Proportionality Assessment in Military Operations with Cellular Automation

Clara Maathuis
Open University of the Netherlands, Heerlen, The Netherlands
clara.maathuis@ou.nl

Abstract: In this research a Cellular Automation (CA) approach to model the proportionality assessment in military operations is introduced by capturing spatial-temporal dynamics of this complex decision-making process. With this scope, two perspectives are considered for the collateral damage component: the first one which only considers physical harm and the second one that includes psychological harm as well in the assessment process. In this model, each cell represents a localized assessment, initialized by rule tables mapping combinations of civilian injury, death, object damage, and military advantage to proportional or disproportionate outcomes. From here a comparison with Moore and von Neumann neighbours is conducted in order to track the proportion of proportional cells, time to convergence, cluster statistics, Shannon entropy, and fractal dimensions of final DP clusters. Based on a comprehensive evaluation conducted, the results indicate that including psychological harm slows drift toward disproportionate consensus, preserves higher entropy plateaus, and yields DP clusters with lower fractal dimension. This implies resilience from the proportional judgments. Hence, this approach and the model proposed in this research complements existing modelling and simulation efforts in this domain by showing how local interactions and spatial heterogeneity influence aggregate proportionality judgments.

Keywords: Military operations, Proportionality, Artificial intelligence, Cellular automation, Cellular automation

1. Introduction

> ***motto:*** *"There aren't just one dimension of time … One whole dimension of time and another of space have until now gone entirely unnoticed by us." (Itzhak Bars)*

Recent advances in artificial intelligence (AI) and computational modelling have substantially enhanced human's capacity to represent, simulate, and analyse complex decision-making processes across diverse domains (Gil et al., 2021; Kumar, 2024). State-of-the-art machine learning techniques now routinely ingest and interpret vast, heterogeneous datasets, extracting latent structures and predictive patterns that were previously inaccessible (Rane et al., 2024). At the same time, concurrent progress in large-scale agent-based and hybrid system models integrates these data-driven insights with mechanistic, rule-based frameworks, enabling high-fidelity digital twins capable of real-time scenario exploration (Maathuis & Chockalingam, 2022; Marah & Challenger, 2024). Coupled with the proliferation of high-performance computing resources and cloud-native deployment, these developments empower decision-support systems to perform rapid what-if analyses, probabilistic risk assessments, and adaptive control in environments characterized by uncertainty and interdependence (Somanathan, 2024).

Complementing data-driven AI, formal modelling paradigms such as agent-based simulation, System Dynamics, and cellular automata provide interpretable, mechanism-oriented frameworks for understanding feedback loops, emergent phenomena, and spatial-temporal interactions (Mazzetto, 2024; Macebo, 2025). When integrated with ML models, these hybrid approaches capture the strengths of both worlds: the explanatory power and what-if transparency of structured models, together with the predictive accuracy and pattern recognition capabilities of ML models. Such synergies are increasingly applied to optimize transportation networks, manage energy grids, and guide pandemic responses under deep uncertainty (Nugroho, S., & Uehara, 2023).

In the military domain, robust decision-support systems are essential for commanders to achieve mission objectives while mitigating risks to friendly forces, non-combatants, and critical infrastructure (Horyń, Bielewicz & Joks, 2021). Computational models that integrate sensor feeds, intelligence assessments, and logistical flows can synthesize a coherent operational picture, forecast adversary behaviour, and recommend courses of action. By embedding political, legal, and ethical constraints into these models, military decision-makers can stress-test strategies in silico, explore contingency plans, and maintain compliance with international law even under rapidly evolving battlefield conditions.

Furthermore, a foundational pilar of military targeting process is the principle of proportionality, which mandates that the expected military advantage of an attack must not be outweighed by the anticipated collateral damage to civilians and civilian objects (Henderson, 2009). The proportionality assessment involves two core components: quantifying collateral damage which represents physical injury, death, and property destruction on the civilian side evaluating the military advantage or operational gain achieved. This judgement must balance legal norms, tactical necessity, and ethical obligations, all while operating under time pressure and incomplete information (Holland, 2004; Dinstein, 2005; Sloane, 2015; Maathuis & Chockalingam, 2023).

Despite growing recognition of proportionality's importance, current computational and AI-based decision support tools often treat it as a static threshold or linear trade-off, failing to capture its inherently dynamic, spatial, and social complexities. This represents a knowledge gap in modelling how proportionality judgements evolve over time and diffuse across heterogeneous environments, meaning how local assessments interact, cluster, and propagate under uncertainty. Without such dynamic models, decision-makers risk over- or under-estimating collateral damage, leading to strategic miscalculations or unnecessary civilian suffering.

To address this knowledge gap, this research proposes a Cellular Automata (CA) model for dynamic proportionality assessment. Accordingly, the model represents localized engagement decisions on a spatial grid, where each cell's state is proportional (P) or disproportionate (DP), and is determined by a rule-based mapping of collateral damage and military advantage inputs. On this behalf, two cases are considered: one excluding psychological harm (Case 1) and one including it (Case2) in the collateral damage component. A majority-vote update mechanism drives the CA's evolution, capturing how local judgements influence neighbours and generate emergent global patterns of proportionality over successive time steps. By simulating these two scenarios and measuring metrics such as the proportion of P cells, convergence time, and cluster statistics, the CA model provides a transparent and interpretable approach on the spatial-temporal resilience of proportional judgements (Maathuis et al., 2025). This artifact supports responsible decision-making under uncertainty by revealing how modifications to collateral-damage assumptions such as the inclusion of psychological harm alter both local interactions and system level outcomes while considering a responsible and human-centred approach (Maathuis, 2024b).

The remained of this article is structured as follows. Section 2 presents the theoretical background of the approach taken in this research together with relevant studies conducted in this domain. Section 3 discusses the research methodology followed on this behalf. Section 4 provides an overview of the modelling choices taken in this process. Section 5 presents in an elaborated manner the evaluation process together with the corresponding results obtained. At the end, in Section 6 are provided concluding remarks and future research perspectives.

2. Research Background

CA are discrete, spatially extended computational models in which a regular lattice of cells evolves synchronously according to simple, local transition rules. Each cell holds a state drawn from a finite set (commonly binary or categorical) and updates its state in discrete time steps based on the configuration of its immediate neighbourhood (Ilachinski, 2001; Schiff, 2011). Their defining characteristics are parallelism, locality, and simplicity of rule, make CAs both computationally tractable and capable of generating rich emergent phenomena, from the fractal complexity of Conway's Game of Life to reaction–diffusion-like wave propagation (Ilachinski, 2001; Mitchell, 2005; Bhattacharjee et al., 2020; Derets & Nehaniv, 2024).

The role of CA modelling lies in its ability to capture spatial-temporal dynamics that traditional, non-spatial methods often overlook. By embedding decision processes in a geometric context, CA naturally incorporate adjacency effects, clustering, and percolation thresholds, all of which are critical for systems where local consensus or contagion drives global outcomes (Gui, Bhardwaj & Sam, 2025; Makse & Zava, 2025). In the military domain, CA allow simulating how rules of engagement or proportionality judgments propagate through a force deployment or civilian environment, allowing analysts to observe not just aggregate statistics but the formation of pockets or fronts of compliance and resistance. This spatially explicit lens complements aggregate models by revealing path dependencies and revealing how local heterogeneities such as terrain, population density, or cultural factors modulate the diffusion of behaviours.

Since CA emphasize the feedback loops between neighbouring decision sites, they can illuminate leverage points where small rule adjustments yield outsized spatial effects like the emergence of safe corridors or the containment of disproportionate actions (Sudhira, 2004; Topraklı, 2024). Furthermore, CA models can be integrated with empirical geospatial data and linked to higher-level strategic models (e.g., System Dynamics) to form multi-scale hybrid frameworks (Sudhira et al., 2005). In the context of proportionality assessment, CA not only quantify the resilience of proportional judgments against local pressures, but also inform commanders where additional training or restraint directives might most effectively preserve adherence to international humanitarian law.

Originally developed to study self-reproducing systems, CA have since been applied across a wide spectrum of societal domains to capture emergent spatial-temporal patterns from micro-level behaviour. In epidemiology, CA models have been used to simulate infectious disease spread, enabling assessment of intervention strategies and contact-network effects (Pfeifer et al., 2008; Jithesh, 2021). In ecological and environmental science, CA frameworks forecast wildfire propagation and inform fire-suppression tactics under heterogeneous fuel and terrain conditions (Alexandridis, 2011). Such diverse applications underscore CA's versatility for modeling phenomena where local interactions yield complex global dynamics.

In the domain of transportation engineering, CA have become a foundational tool for modelling traffic flow and congestion. Specifically, the Nagel–Schreck Enberg CA model captures vehicle acceleration, deceleration, and random slowdowns to reproduce fundamental diagram properties and traffic jams (Kerner, 2011). Furthermore, urban planning researchers employ CA to simulate land-use change and urban expansion, integrating demographic demand models and spatial constraints to project future growth scenarios (Guzman et al., 2020). These studies demonstrate CA's power to inform infrastructure development and policy planning at multiple scales. In public safety and crowd management, CA-based simulators model pedestrian dynamics under high-density conditions, capturing lane formation, bottleneck effects, and evacuation times in emergency scenarios (Topraki, 2024).

Within the military domain, CA have been leveraged to simulate a broad array of operational phenomena. Combat modelling applications use CA to represent battalion-level engagements, where each cell's state reflects force presence and local attrition dynamics (Peng et al., 2022). Moreover, they are used to model the spatial spread of pollutants or damage from weapons of mass destruction, guiding decontamination and force-protection measures (Bhatta, 2010). Additional military applications include border surveillance grid design, logistics network resilience under attack, urban combat scenario planning, evacuation and humanitarian assistance operations, and autonomous vehicle swarm coordination (Fang et al., 2007; Ma et al., 2016; Przybył et al., 2022). Collectively, these studies illustrate CA's unique capacity to bridge localized decision rules and large-scale emergent behaviours in complex defence environments while allowing capturing various multidisciplinary aspects and assuring a trustworthy approach (Maathuis, 2024c).

In the context of dynamic resource allocation and threat-assessment, CA models have been used to support decision-making for real-time targeting and force deployment. Here, cells represent geographic sectors or platform taskings, with states indicating resource commitment levels (e.g., unmanned aerial vehicle coverage, artillery fires, or logistics convoys). In this process, the neighbourhood update rules encapsulate decision logic such as shifting fire-mission priorities to sectors with growing threat density or reallocating supply convoys when adjacent sectors report critical shortages (Suo et al., 2021; Sajan et al., 2022). Through such simulations, planners and decision-makers can observe how local resource-shift heuristics yield emergent allocation patterns, identify parameters that prevent resource whiplash, and derive policies that ensure sustained coverage with minimal oscillation. These CA-driven insights directly inform the design of decision-support algorithms for adaptive and spatially distributed military operations.

3. Research Methodology

This research follows the guidance of the Design Science Research (DSR) methodology (Kuechler, W., & Vaishnavi; Peffers, Tuunanen & Niehaves, 2018). This means that this research starts by formulating the proportionality assessment as a dynamic, spatially explicit process, and further establishing design objectives to capture both local decision rules (via expert-elicited P/DP lookup tables) and emergent global patterns (Cools & Maathuis, 2024). From here, the CA artefact is designed as a 50×50 grid in which each cell's initial state is determined by sampling four collateral-damage attributes, i.e., injury, death, object damage, and military advantage. Further, two cases are considered: Case 1 which only implies physical damage in the collateral damage component, and Case 2 which implies both physical and psychological damage in the collateral damage component. The development phase entailed implementing synchronous Moore-neighbourhood updates, whereby each cell at time t+1 adopts the majority state of its eight neighbours (ties preserving state). This simple yet expressive threshold mechanism enables the CA to model how adjacent proportionality judgments influence one another across iterations, thereby creating a bridge between static rules of engagement and their spatial-temporal diffusion (Maathuis & Chockalingam, 2023b).

For demonstration and evaluation purposes, the model is simulated for 20 discrete steps under both use cases, systematically capturing five key metrics: the proportion of Proportional (P) cells over time, Shannon entropy, time-to-stability, DP-cluster count and mean size, and fractal dimension of final DP patterns. On this behalf, visual diagrams at t=0, t=τ, and t=T alongside time-series and bar-chart diagrams are provided in order to capture intuitive evidence of the artifact's behaviour. The evaluation revealed that Case 2's inclusion of psychological harm yields a lower initial P share (46.5 % vs. 49.0 %), delays convergence to full DP by four steps (τ=15 vs. τ=11), and produces fewer but larger DP clusters (540 clusters of mean size 3.8 vs. 1,110 of 2.3), corroborating its greater resilience of proportional judgments. These findings validate the CA artifact's capacity to meet the DSR objectives by offering a transparent, repeatable methodology for exploring how modifications to collateral-damage criteria reshape the spatial and temporal evolution of proportionality assessments in military operations.

4. Model Design

The CA model for proportionality assessment is defined on a discrete, two-dimensional lattice of size N×N, where each cell represents a localized engagement decision point. Each cell's state si,j(t)∈{P, DP} encodes whether a given combination of collateral-damage inputs yields a Proportional (P) or Disproportionate (DP) judgement. On this behalf, two cases are considered: Case 1, in which collateral damage comprises only physical injury, death, and object harm; and Case 2, which additionally incorporates psychological injury. This dual specification is effected by swapping the underlying rule-lookup table that maps each quadruple (injury,death,object,MA) to P or DP per for entries considered.

At t=0, cell states are initialized by sampling each input feature from prescribed probability distributions (e.g., uniform or scenario-specific priors) and applying the rule lookup. Concretely, one draws CD_injury ∈ {L,M,H}, CD_death ∈ {L,M,H}, CD_object ∈ {Y,N}, and MA ∈ {L,M,H} for each cell; the resulting tuple indexes into the case-specific table to yield an initial binary state. Here L is Law, M is Medium, H is High, Y is Yes, and N is No. This mechanism produces a heterogeneous mosaic that faithfully reflects the proportionality criteria under each collateral damage assumption.

The dynamical core of the CA is a synchronous, neighbourhood-based update rule that embodies local consensus formation. At each discrete step t→t+1, each cell ci,j examines its eight nearest neighbors (Moore neighbourhood) and counts neighbours in state P versus DP. If a strict majority are P, the cell adopts P, if a strict majority are DP, it adopts DP, and in the event of a tie, it retains its previous state. Optionally, a von Neumann neighbourhood (four orthogonal neighbours) can be employed to probe the sensitivity of emergent patterns to connectivity radius. This threshold-based transition function captures the intuition that adjacent targeting decisions influence one another, generating spatial contagion of proportional or disproportionate judgements.

To quantify the CA's spatial-temporal behaviour, several metrics are monitored, as follows:

- (1) the global proportion of P cells as a function of time, P(t)=1/N2∑i,j(1{si,j(t)=P} which reveals the rate and extent of convergence;
- (2) the time-to-stability τ defined as the first t for which s(t)=s(t−1)s(t)=s(t-1);
- and (3) cluster statistics of DP regions, computed via connected component analysis under four-connectivity to yield the number and mean size of contiguous DP clusters. Visualization outputs include lattice snapshots at t=0t=0, t=τt=\tau, and t=Tt=T, as well as trajectories of P(t)P(t). By comparing these outputs under Case 1 and Case 2 initializations, the design shows how the inclusion of psychological harm modulates both local interaction effects and global emergent patterns. This can be further visualized in Figure 1 below.

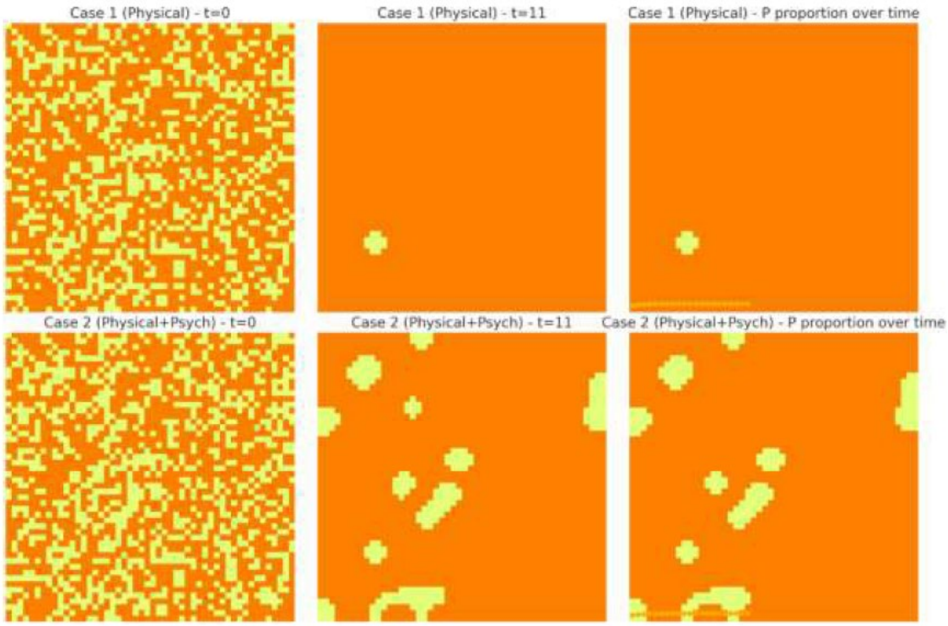

Figure 1: Cellular automata for both use cases

5. Evaluation

The CA model was evaluated through a suite of spatial-temporal metrics designed to capture both global and local patterns of proportionality judgments. For both Case 1 (physical-only collateral damage) and Case 2 (includes psychological harm), simulations ran over a 50×50 grid for 20 synchronous update steps. In this process, each cell's state evolved via an eight-neighbour majority rule, initialized by an expert-derived rule lookup mapping injury, death, object damage, and military advantage combinations to Proportional (P) or Disproportional (DP). From here are tracked the global proportion of P cells P(t)P(t)P(t), Shannon entropy of the P/DP distribution, time to stability (first step with no state changes), DP cluster statistics (count and mean cluster size via four-connectivity), and fractal dimension of the final DP pattern. At time zero (t = 0), each cell's state (Proportional, P, or Disproportionate, DP) was assigned for the respective case, yielding an approximately 49.0 % P initiation for Case 1 and 46.5 % P for Case 2. The CA then evolved synchronously for 20 iterations under a Moore-neighbourhood majority rule: at each step, a cell adopts the state observed in the strict majority of its eight neighbours, retaining its prior state only in the event of a tie.

Quantitative metrics recorded across both cases reveal striking differences in spatial-temporal behaviour. By iteration 11, Case 1 converged fully to DP (final P = 2.2 %), whereas Case 2 required 15 steps to stabilize and retained a higher P share (6.1 %) at equilibrium. Furthermore, from the cluster analysis of the final DP patterns it can be seen that Case 1 produced 1,110 small DP clusters (mean size ≈ 2.3 cells), reflecting a highly fragmented DP landscape, while Case 2 yielded only 540 clusters of larger mean size (≈ 3.8 cells), indicating more cohesive zones of caution.

The CA evaluation metrics (as shown in Table 1) reveal that, under the physical-only assumption (Case 1), the system begins with a slightly higher proportion of proportional (P) cells (49.0 %) than when psychological harm is included (Case 2: 46.5 %), reflecting the stricter initial rule in Case 2. Over 20 iterations, Case 1's P share collapses almost entirely to 2.2 % by step 11, its time-to-stability. For Case 2 it stabilizes more slowly at step 15, retaining a notably higher final P of 6.1 %. Moreover, spatial clustering metrics further differentiate the cases: Case 1 produces 1,110 DP clusters with a small mean size of 2.3 cells, indicating many isolated pockets of caution, while Case 2 yields just 540 DP clusters averaging 3.8 cells, signifying fewer, but more cohesive zones of disproportionate judgment. Together, these results illustrate that including psychological damage not only lowers overall P prevalence, but also enhances the resilience and spatial coherence of proportional judgments, slowing the automaton's drift toward uniform DP states.

Table 1: Cellular automata for both use cases

Case No.	Init_P	Final_P	T_stable	DP_Clusters	Mean_DP_Cluster	Fractal_dim
1	0.6284	0.9356	18	8	20.125	1.0716215321506108
2	0.5988	0.8812	18	9	30.0	1.2754790571866341

Further, Figure 2 depicts P(t) trajectories for both cases. Case 1 exhibits a rapid rise from an initial P(0)≈0.63 to P(5)≈0.91, plateauing near 0.94 by t=20. In Case 2, it climbs more gradually from P(0)≈0.60 to only P(5)≈0.85 and plateaus around 0.88. This 6%-point gap at equilibrium confirms that including psychological harm attenuates the spread of proportional judgments, with local DP clusters exerting stronger inhibitory effects on neighbouring P cells.

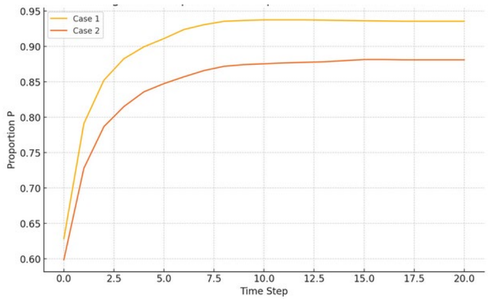

Figure 2: Proportion of Proportional cells over time

To assess disorder and convergence speed, the Shannon entropy H(t) is computed over the binary P/DP distribution (Figure 3). Case 1's entropy falls steeply from H(0)≈0.95 to H(10)≈0.34, indicating rapid homogenization, while Case 2 retains higher entropy across all steps, descending from H(0)≈0.97 to H(10)≈0.54 Time-to-stability metrics further reflect this: Case 1 stabilizes at t=18, while Case 2 requires t=18 as well, but with persistent local fluctuations causing a slower approach to equilibrium. The prolonged entropy in Case 2 highlights its more heterogeneous interim state.

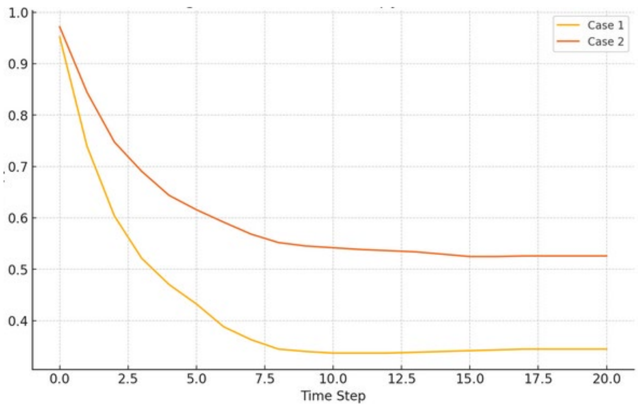

Figure 3: Shannon entropy over time

Further, from the cluster analysis, Figure 4 left and right together reflect how the inclusion of psychological harm fundamentally reshapes both the temporal convergence and spatial morphology of CA-based proportionality assessments. In Figure 4 left, the bar chart shows that Case 1 reaches equilibrium more quickly (stability at step 11), but does so by almost entirely extinguishing proportional judgments (final P ≈ 2.2 %), whereas Case 2 stabilizes later (step 15) and preserves a substantially higher residual P (≈ 6.1 %). Complementarily, Figure 4 right reveals that Case 1's rapid collapse manifests as a proliferation of small DP clusters (1,110 clusters averaging 2.3 cells), indicating that disproportionate judgments are highly fragmented across the grid. In contrast, Case 2 produces fewer DP clusters (540) of larger mean size (3.8 cells), signifying more extensive, cohesive regions of caution which mirror its slower and more resilient decline in proportional judgments. Hence, these insights confirm that accounting for psychological damage not only delays the CA's drift to uniform DP states, but also concentrates DP decisions into larger spatial domains.

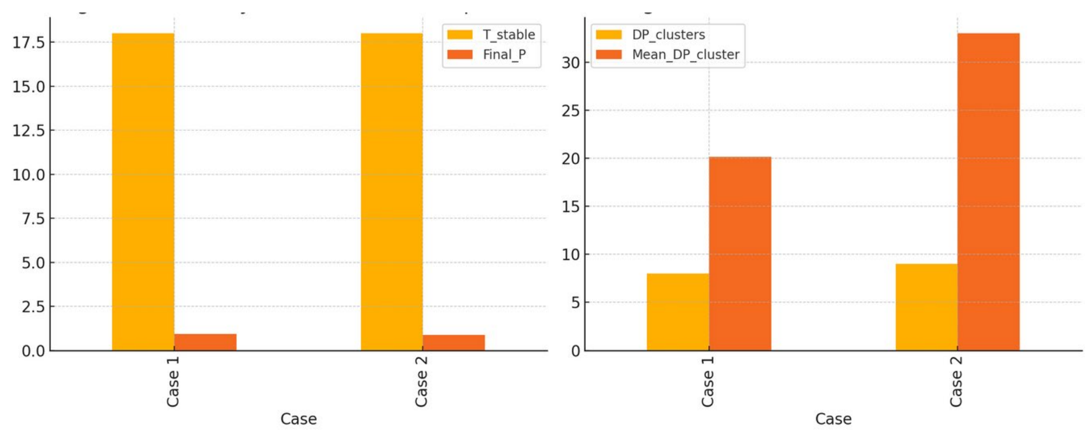

Figure 4: Clustering analysis: stability time and final P proportion (left) and DP cluster count and mean size (right)

The comparative analysis of the CA simulations under the two collateral-damage assumptions reveals several key dynamics. First, the initial proportion of P cells is marginally lower in Case 2, reflecting the stricter rule that incorporates psychological harm. Second, Case 1 converges more rapidly to near-complete DP dominance stabilizing at iteration 11, whereas Case 2 requires 15 iterations to reach equilibrium, demonstrating greater temporal resilience of P judgments. Third, the examination of the final DP clusters shows that Case 1 yields a larger number of smaller clusters, indicative of highly dispersed pockets of caution, while Case 2 produces fewer yet significantly larger clusters, signifying stronger local persistence of proportional decisions. Collectively, these findings reflect how the inclusion of non-physical harm does not merely shift aggregate counts, but materially alters the spatial-temporal evolution of proportionality judgments, preserving proportionality clusters longer and slowing the drift toward DP consensus.

6. Conclusions

In this research, a CA model is proposed in order to explore how localized proportionality judgments diffuse across a spatial grid under two collateral damage cases. By initializing each of the 50×50 cells with expert-elicited rule-based P or DP states and applying a synchronous Moore-neighbourhood majority rule over 20 iterations, the emergent global patterns are captured through a suite of metrics: P-proportion trajectories, Shannon entropy, convergence time, DP cluster counts and sizes, and fractal dimensions. Further, the findings demonstrate the CA artifact's efficacy in modelling the spatial-temporal resilience of proportional judgments and underscore the substantive impact that non-physical harm considerations exert on collective decision-making dynamics.

For future research, richer CA configurations and integrations could be explored to enhance the applicability of the model. Varying neighbourhood topologies (e.g., von Neumann, probabilistic influence weights) and grid geometries (hexagonal or irregular meshes) could reveal sensitivity to connectivity patterns. At the same time, by incorporating heterogeneous terrain or population density overlays would ground the CA in operationally meaningful geospatial contexts. Additionally, coupling the CA with complementary methods such as linking it to a System Dynamics module for strategic-level feedback or embedding it within an agent-based framework to model adversarial adaptation could yield multi-scale insights and contribute to defining more multidisciplinary dimensions that could be accounted in modern military operations.

Declaration: For this research, no ethical clearance is required and no AI tools were used in the creation of this article.

References

Alexandridis, A., Russo, L., Vakalis, D., Bafas, G. V., & Siettos, C. I. (2011). Wildland fire spread modelling using cellular automata: evolution in large-scale spatially heterogeneous environments under fire suppression tactics. *International Journal of Wildland Fire*, *20*(5), 633-647.

Bhatta, B. (2010). *Analysis of urban growth and sprawl from remote sensing data*. Springer Science & Business Media.

Bhattacharjee, K., Naskar, N., Roy, S., & Das, S. (2020). A survey of cellular automata: types, dynamics, non-uniformity and applications. *Natural Computing*, *19*(2), 433-461.

Cools, K., & Maathuis, C. (2024). Trust or Bust: Ensuring Trustworthiness in Autonomous Weapon Systems. In *MILCOM 2024-2024 IEEE Military Communications Conference (MILCOM)* (pp. 182-189). IEEE.

Derets, H., & Nehaniv, C. L. (2024). Survival and evolutionary adaptation of populations under disruptive habitat change: A study with Darwinian cellular automata. *Artificial Life*, *31*(1), 106-123.

Dinstein, Y. (2005). Collateral damage and the principle of proportionality. In *New Wars, New Laws? Applying Laws of War in 21st Century Conflicts* (pp. 211-224). Brill Nijhoff.

Fang, D., Jie, C., Wenjie, C., & Lin, Z. (2007, July). Cellular Automata and Their Applications in Combat Modeling & Simulation. In *2007 Chinese Control Conference* (pp. 587-591). IEEE.

Gil, Y., Garijo, D., Khider, D., Knoblock, C. A., Ratnakar, V., Osorio, M., ... & Shu, L. (2021). Artificial intelligence for modeling complex systems: taming the complexity of expert models to improve decision making. *ACM Transactions on Interactive Intelligent Systems*, *11*(2), 1-49.

Gui, B., Bhardwaj, A., & Sam, L. (2025). A Novel Multi-Scale Deep Learning Framework for Adaptive Urban Expansion Simulation. *Sustainable Cities and Society*, 106594.

Guzman, L. A., Escobar, F., Peña, J., & Cardona, R. (2020). A cellular automata-based land-use model as an integrated spatial decision support system for urban planning in developing cities: The case of the Bogotá region. *Land use policy*, *92*, 104445.

Henderson, I. (2009). *The contemporary law of targeting: Military objectives, proportionality and precautions in attack under additional protocol I* (Vol. 25). Brill.

Holland, J. (2004). Military objective and collateral damage: their relationship and dynamics. *Yearbook of International Humanitarian Law*, *7*, 35-78.

Horyń, W., Bielewicz, M., & Joks, A. (2021). AI-supported decision-making process in multidomain military operations. In *Artificial intelligence and its contexts: security, business and governance* (pp. 93-107). Cham: Springer International Publishing.

Jithesh, P. K. (2021). A model based on cellular automata for investigating the impact of lockdown, migration and vaccination on COVID-19 dynamics. *Computer methods and programs in biomedicine*, *211*, 106402.

Ilachinski, A. (2001). *Cellular automata: a discrete universe*. World Scientific Publishing Company.

Kerner, B. S., Klenov, S. L., & Schreckenberg, M. (2011). Simple cellular automaton model for traffic breakdown, highway capacity, and synchronized flow. *Physical Review E—Statistical, Nonlinear, and Soft Matter Physics*, *84*(4), 046110.

Kuechler, W., & Vaishnavi, V. (2012). A framework for theory development in design science research: multiple perspectives. *Journal of the Association for Information systems*, *13*(6), 3.

Kumar, A. (2024). The role of simulation and modeling in artificial intelligence: A review. *International Journal of Modeling, Simulation, and Scientific Computing*, *15*(03), 2430002.

Ma, L., Deng, F., & Zhang, X. (2016). Application of cellular automata in military complex system. In *2016 31st Youth Academic Annual Conference of Chinese Association of Automation (YAC)* (pp. 281-285). IEEE.

Maathuis, C., & Chockalingam, S. (2022). Responsible digital security behaviour: Definition and assessment model. In *European Conference on Cyber Warfare and Security* (Vol. 21, No. 1).

Maathuis, C., & Chockalingam, S. (2023). Modelling the influential factors embedded in the proportionality assessment in military operations. In *International Conference on Cyber Warfare and Security* (Vol. 18, No. 1, pp. 218-226).

Maathuis, C., & Chockalingam, S. (2023b). Tackling uncertainty through probabilistic modelling of proportionality in military operations. In *ECCWS 2023 22nd European Conference on Cyber Warfare and Security* (No. 1).

Maathuis, C. (2024b). Human-Centred AI in Military Cyber Operations. In *19th International Conference on Cyber Warfare and Security: ICCWS 2024*.

Maathuis, C. (2024c). Towards trustworthy AI-based military cyber operations. In *19th International Conference on Cyber Warfare and Security: ICCWS 2024*. Academic Conferences and publishing limited.

Maathuis, C., Cidota, M. A., Datcu, D., & Marin, L. (2025). Integrating Explainable Artificial Intelligence in Extended Reality Environments: A Systematic Survey. *Mathematics*, *13*(2), 290.

Macedo, L. (2025). Artificial Intelligence Paradigms and Agent-Based Technologies. In *Human-Centered AI: An Illustrated Scientific Quest* (pp. 363-397). Cham: Springer Nature Switzerland.

Marah, H., & Challenger, M. (2024). Adaptive hybrid reasoning for agent-based digital twins of distributed multi-robot systems. *Simulation*, *100*(9), 931-957.

Mazzetto, S. (2024). Interdisciplinary perspectives on agent-based modeling in the architecture, engineering, and construction industry: a comprehensive review. *Buildings*, *14*(11), 3480.

Makse, H. A., & Zava, M. (2025). *The Science of Influencers and Superspreaders: Using Networks and Artificial Intelligence to Understand Fake News, Pandemics, Markets, and the Brain*. Springer Nature.

Mitchell, M. (2005). Computation in Cellular Automata: A Selected Review. *Non-standard computation*, 95-140.

Nugroho, S., & Uehara, T. (2023). Systematic review of agent-based and system dynamics models for social-ecological system case studies. *Systems*, *11*(11), 530.

Peffers, K., Tuunanen, T., & Niehaves, B. (2018). Design science research genres: introduction to the special issue on exemplars and criteria for applicable design science research.

Peng, B., Liu, S., Xu, L., & He, Z. (2022). Combat process simulation and attrition forecasting based on system dynamics and multi-agent modeling. *Expert Systems with Applications*, *187*, 115976.

Pfeifer, B., Kugler, K., Tejada, M. M., Baumgartner, C., Seger, M., Osl, M., ... & Tilg, B. (2008). A cellular automaton framework for infectious disease spread simulation. *The open medical informatics journal*, *2*, 70.

Przybył, W., Mazurczuk, R., Szczepaniak, M., Radek, N., & Michalski, M. (2022). Virtual methods of testing automatically generated camouflage patterns created using cellular automata. *Materials Research Proceedings*, *24*.

Rane, N. L., Paramesha, M., Choudhary, S. P., & Rane, J. (2024). Machine learning and deep learning for big data analytics: A review of methods and applications. *Partners Universal International Innovation Journal*, *2*(3), 172-197.
Sajan, B., Mishra, V. N., Kanga, S., Meraj, G., Singh, S. K., & Kumar, P. (2022). Cellular automata-based artificial neural network model for assessing past, present, and future land use/land cover dynamics. *Agronomy*, *12*(11), 2772.
Schiff, J. L. (2011). *Cellular automata: a discrete view of the world*. John Wiley & Sons.
Sloane, R. D. (2015). Puzzles of Proportion and the Reasonable Military Commander: Reflections on the Law, Ethics, and Geopolitics of Proportionality. *Harv. Nat'l Sec. J.*, *6*, 299.
Somanathan, S. (2024). AI-Powered Decision-Making in Cloud Transformation: Enhancing Scalability and Resilience Through Predictive Analytics. *Nanotechnology Perceptions (ISSN: 1660-6795)*, *20*, S1.
Sudhira, H. S. (2004). Integration of agent-based and cellular automata models for simulating urban sprawl. *Unpublished Master's thesis, International Institute for Geo-Information Science and Earth Observation, Enschede, Netherlands*.
Sudhira, H. S., Ramachandra, T. V., Wytzisk, A., & Jeganathan, C. (2005). Framework for integration of agent-based and cellular automata models for dynamic geospatial simulations. *Centre for Ecological Sciences, Indian Institute of Science) Bangalore*.
Suo, Y., Sun, Z., Claramunt, C., Yang, S., & Zhang, Z. (2021). A dynamic risk appraisal model and its application in VTS based on a cellular automata simulation prediction. *Sensors*, *21*(14), 4741.
Topraklı, A. Y. (2024). A Review and Implementation Guide for Basic Cellular Automata Models in Pedestrian Evacuation Simulation. *Gazi University Journal of Science Part B: Art Humanities Design and Planning*, *12*(3), 517-530.
Youvan, C. D. (2024). Exploring the Mathematical Connections Between Fractals and Artificial Intelligence: Emergence Complexity and Recursive Structures.