Pedagogically Asymmetric Elo Models for Multidimensional Adaptive Learning Systems

David Janda, Antonín Jančařík, Jakub Michal and Tomáš Jeřábek
Charles University, Faculty of Education, Prague, Czech Republic

david.janda@pedf.cuni.cz
antonin.jancarik@pedf.cuni.cz
jakub.michal@pedf.cuni.cz
tomas.jerabek@pedf.cuni.cz

Abstract: Adaptive learning systems commonly use psychometric models to estimate learner ability and task difficulty. Among the most widely used approaches are Item Response Theory (Harvey & Hammer, 1999) and modifications of the Elo rating system (Pelánek, 2016). These models typically assume a symmetric learner–task interaction, with maximum diagnostic information occurring at approximately a 50% probability of success. While appropriate for testing, this equilibrium may not correspond to pedagogically desirable conditions for sustained learning. This article presents a pedagogically asymmetric parameterisation of a multidimensional Elo model for adaptive mathematics education. The model combines a multidimensional representation of mathematical competencies based on the Global Proficiency Framework for Mathematics (UNESCO et al., 2020), scaffold-aware partial-credit scoring, and separate equilibrium parameters for learner and task updates. This enables learner-profile development and task-difficulty dynamics to be controlled differently rather than being governed by the same symmetric equilibrium. The model was implemented in the Edu-AI system for preparing Czech elementary school learners for standardised mathematics entrance examinations (Jančařík et al., 2023). The pilot dataset comprised 63,020 learner–task interactions involving 628 learners and 233 tasks. A replay-based simulation preserved the chronological interaction histories, achieved scores, and scaffold use while recalculating learner and task parameters under iteratively adjusted equilibrium settings. The analysis focused on model stability, learner-profile dynamics, task-difficulty convergence, and the interaction between equilibrium settings and scaffold-aware scoring. The replay results showed that relatively small changes in equilibrium parameters substantially affect learner and task dynamics. A configuration with an equilibrium of 0.80 for learner updates and 0.85 for task updates produced stable and pedagogically interpretable behaviour under the scoring policy used in the pilot. These values are treated as promising initial settings rather than as optimal parameters for learning effectiveness. The study therefore demonstrates the feasibility of introducing pedagogical asymmetry into a multidimensional Elo model and provides a basis for prospective testing of the model's educational effects.

Keywords: Adaptive Learning, Elo Rating System, Multidimensional Learner Modelling, Mathematics Education, Educational AI, Learning Analytics

1. Introduction

Adaptive learning systems represent a significant area of current research in educational technology. Many current systems utilise psychometric models based on Item Response Theory (IRT) (Harvey & Hammer, 1999; Cai et al., 2016; Chen et al., 2025), modifications of the Elo rating system (Pelánek, 2016; Vermeiren et al., 2025)—originally developed for evaluating chess players—or combinations thereof (Antal, 2013). These approaches enable the continuous modelling of a student's abilities and the dynamic adjustment of task difficulty.

However, most adaptive systems are primarily focused on the accuracy of measuring student performance. Optimal diagnostic value occurs when the probability of correctly solving a task is approximately 50%, which, among other things, determines the selection of appropriate subsequent tasks for a specific student. This principle corresponds to maximising the informational value of the learner-task interaction, but not necessarily to optimising the conditions for the student's learning, which is generally not considered in testing.

When the primary goal of the learner-task interaction is to support the student's learning through problem solving, the situation is different. From a pedagogical perspective, it can be assumed that effective learning occurs more in situations of manageable challenge, where the student solves tasks with a relatively high probability of success while remaining exposed to appropriate cognitive demands (de Bruin et al., 2023). From this perspective, it is important for an adaptive system to keep tasks within the realm of manageable challenges, rather than prolonged failure.

This paper builds upon the previous development of an AI assistant for mathematics education presented at this conference (Jančařík et al., 2022). The original system focused primarily on chatbot-supported student problem-

solving. It used pre-prepared sequences of problems and supporting materials, along with simple adaptive mechanisms based on difficulty selection and navigation between thematic units.

The current version of the system represents a further step toward a fully adaptive multidimensional learner model that utilises a pedagogically modified Elo rating system, scaffold-aware scoring, and a curriculum-oriented representation of mathematical competencies.

The proposed model shifts the emphasis from maximising diagnostic accuracy toward supporting pedagogically appropriate learning conditions. The main objective of this article is to present a pedagogically asymmetric parameterisation of the Elo system and examine its behaviour using empirical learner–task interaction data and replay-based simulation. The study addresses two research questions:

RQ1: Can a pedagogically asymmetric Elo model, using separate equilibrium parameters for learner and task updates, produce dynamics consistent with the intended pedagogical behaviour?

RQ2: Which learner and task equilibrium parameter settings appear to provide a stable and pedagogically interpretable balance between learner-profile development and task-difficulty dynamics under scaffold-aware scoring? The article presents four key innovations of the proposed model:

1. pedagogically asymmetric parameterisation of the Elo system,
2. multidimensional representation of mathematical competencies,
3. scaffold-aware scoring that reflects the use of support mechanisms,
4. a stabilised mechanism for updating task difficulty.

2. Theoretical Background

2.1 Elo Models in Education

The Elo rating system was originally developed to evaluate the performance of chess players (Elo, 1978). It began to be used in educational settings primarily because of its simplicity, continuous parameter updates, and low computational requirements. In educational applications, the Elo model is typically interpreted similarly to IRT models: a student's ability and the difficulty of a task are represented on a single scale, and the probability of success is derived from the difference between these values (Pelánek, 2016).

Compared to classical IRT models, the Elo model allows for simpler continuous parameter updates and better supports dynamic learner modelling in real time (Pelánek, 2016; Abdi et al., 2019). This is particularly important in adaptive learning environments, where the system continuously responds to the student's current performance.

Traditional Elo models implicitly assume a symmetrical system. If the difficulty of a task matches the student's level, the expected probability of success is approximately 50%. This principle is suitable for rating subjects that interact directly with one another (e.g., in performance evaluation, such as chess) and for diagnostic and testing environments where, through interaction with tasks, direct comparisons among students are possible. However, the target expected score of 50% may not align with the pedagogical requirements for effective learning.

2.2 Adaptive Learning and Desirable Difficulty

Current research in adaptive learning emphasises the importance of personalisation, scaffolding, and long-term learner support. Adaptive systems no longer serve merely to diagnose performance but are increasingly focused on supporting learning, motivation, and the long-term development of competencies (VanLehn, 2011; Walkington & Bernacki, 2018).

Educational theories of desirable difficulty assume that effective learning through problem-solving occurs in situations involving manageable challenges. A task should be challenging enough to support the development of competencies, yet it must not lead to the learner's long-term failure (Bjork & Bjork, 2020; de Bruin et al., 2023).

From a learning perspective, the explainability and interpretability of learner models also play a significant role in the interaction between the student and the task (Bull & Kay, 2016; Jiang, 2025). It is important for both teachers and students that the system not only provides a final score but also enables students to reflect on the outcome of their interaction with the task and interpret their strengths and weaknesses in specific areas.

Alternative approaches to adaptive learner modelling pursue related but distinct objectives. Knowledge-tracing models use previous learner interactions to estimate evolving knowledge states and predict future performance (Liu et al., 2025). Adaptive task-difficulty models may instead regulate task difficulty around a predefined target performance; for example, Kassenkhan et al. (2026) use a target mastery rate of 0.75. The present approach differs in that it introduces separate equilibrium parameters for learner and task updates, allowing their dynamics to be adjusted asymmetrically.

2.3 Previous Development of the System

The proposed model builds on the AI Assistant for Students and Teachers project, which focuses on supporting mathematics and the Czech language through AI technologies. The original version of the system combined a chatbot interface, thematic exercise sequences, instructional videos, and simple adaptive mechanisms. The system was implemented as both a web-based application and a Telegram app.

The pilot implementation revealed a need for more personalised learning, more detailed learner modelling, and more precise task adaptation based on the student's specific mathematical competencies. These requirements led to the design of the multidimensional Elo model presented in this article. At the same time, gamification elements are being integrated into the system, and a chat system is being developed to facilitate direct discussions about the task being solved.

3. System Design

3.1 Introduction to the Edu-AI System

The Edu-AI system (Figure 1) is an adaptive learning environment designed to prepare Czech elementary school students for the standardised mathematics entrance exams. The system combines pre-designed learning pathways with a continuously updated learner model based on a multidimensional Elo vector.

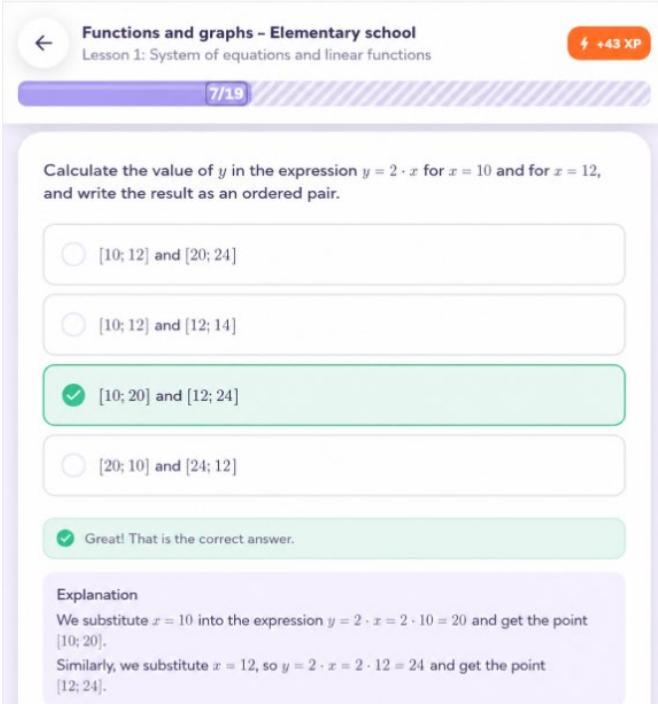


Figure 1: System environment

The pilot implementation of the system was designed as a guided, adaptive environment that combines curriculum-structured instruction and personalised practice. All students first take an initial diagnostic test to establish the baseline learner profile. The result of this test is an initial multidimensional Elo vector representing the student's current level of mathematical competence across the components of the GPF framework. Initial task parameters were estimated using AI-assisted matching to analogous examples and competency descriptions in the Global Proficiency Framework and subsequently reviewed by members of the research team with expertise in mathematics education.

Following an initial assessment, students proceed through eight thematically organised lessons focused on areas of mathematics aligned with the requirements of the standardised entrance exams in the Czech Republic. At this stage, instruction is guided by pre-prepared content that combines:

- instructional materials,
- illustrative examples,
- scaffolded tasks,
- practice activities.

The aim of this section is not only to assess the student's performance but also to gradually modify the learner profile through interactions with tasks of varying relevance and difficulty. Each completed task updates the individual components of both the task's and the student's Elo vector based on:

- correctness of the solution,
- use of hints,
- relevance of the task to specific competencies,
- estimated difficulty of the task.

After completing the guided portion of the lesson, the system proceeds to a short practice session related to the lesson and, if necessary, to an adaptive practice phase based on a database of exercises.

In the current version, all tasks are multiple-choice and include the question, answer options (with feedback for incorrect answers), corresponding scores, complete solutions, two levels of hints (a brief hint and a more detailed hint), and the metadata required for implementation in the application. The selection of subsequent tasks for any additional practice beyond the scope of individual lessons is determined dynamically by the current distance between the learner's profile and the task profile.

At the same time, the entire system strives to maintain a pedagogically defined balance between student success and the cognitive difficulty of the tasks. The resulting adaptive learning path combines guided instruction, personalised practice, scaffolded support, and real-time learner-model updates.

3.2 Multidimensional Representation of Competencies

The system uses a multidimensional model based on the Global Proficiency Framework for Mathematics (GPF). The pilot implementation uses a 35-dimensional representation of mathematical competencies corresponding to the individual subconstructs of the GPF. An Elo vector, therefore, represents each student's profile:

$$L = (l_1, l_2, \dots, l_{35})$$

In accordance with this approach, each task is described by a 35-dimensional difficulty vector D, which expresses the difficulty of the task in the context of individual competencies, consisting of 35 components with values on a scale of 1 to 10, as well as a 35-dimensional relevance vector expressing the relevance of the task to individual competencies with values ranging from 0 to 2 (0 = no relationship, 1 = marginal relevance, 2 = significant relevance).

Task classification was performed using AI-assisted classification followed by human validation.

3.3 Pedagogically Asymmetric Parameterisation of the Elo Model

3.3.1 The Classic Elo Model

As a starting point, we used the basic Elo rating formula to calculate each category's Elo rating:

$$E = \frac{1}{1+e^{-\lambda(L-D)}}.$$

It works in such a way that when $L = D$ the value of E is 0.5. Thus, the student is expected to have a success rate of approximately 50%. The parameter λ influences the resulting value of E when $L \neq D$ and determines the impact of a specific interaction on the subsequent classification of the task.

The new value of L' (or D') is then determined using the following equation:

$$L' = L + K_n \cdot (S - E),$$

$$D' = D - K_m \cdot (S - E),$$

where S is the actual score, E is the expected score, and K_n is a function that limits the magnitude of the change depending on the number of student interactions n (or task m), defined as follows:

$$K_n = \frac{K_0}{1 + \alpha n}.$$

Given the 0–10 scale used to describe the student's profile, $K_o = 1$ and $\alpha = 0.1$ were selected.

3.3.2 Scaffold-Aware Scoring

The current version of the app contains only multiple-choice questions, intended to demonstrate and explore the effects of the scaffold-aware scoring system, one of the pillars of the pedagogically oriented Elo model. Unlike binary scoring or a three-point chess rating system, this scoring system considers the use of hints:

- – correct solution without assistance,
- 0.75 – correct solution with basic hints,
- 0.5 – correct solution with additional hints,
- 0 – incorrect solution.

The original model design assumed a pedagogical balance approaching 0.9, which corresponds to the intuitive notion of high student performance on multiple-choice tasks in adaptive learning.

In a scaffold-aware scoring environment, this configuration led to an excessive decline in learner profiles. The reason was that a significant portion of interactions did not yield a binary score of 1.0 but rather values of 0.75 or 0.5, reflecting the use of hints. The effective distribution of learner scores thus shifted toward lower values, suggesting the model needed adjustment.

3.4 Modifications to the System for Replay-Based Simulation

To perform replay-based simulations, a system modification was designed to reflect the scaffold-aware scoring described above while also meeting the required pedagogical asymmetry (from the student's perspective, it is desirable to present challenges with a theoretically lower expected score than from the task's perspective). To adjust the student's Elo rating, we therefore use:

$$E_L = \frac{1}{1 + e^{-(\lambda(L-D)+b_L)}}.$$

Similarly, to calculate the Elo rating of a task, we use:

$$E_D = \frac{1}{1 + e^{-(\lambda(L-D)+b_D)}}.$$

The purpose of the replay-based simulation is to test appropriate settings for the b_L and b_D parameters to meet the requirements described above when applied to the obtained dataset.

3.5 Testing Process and Testing Dataset

An introductory diagnostic test was designed for the app, followed by a total of eight courses covering basic algebraic expressions, ratios and proportionality, linear functions, equations and inequalities, measurements in geometry, the basics of statistics, descriptive geometry, and fractions. The courses were released sequentially in the order listed, one week apart. All participating teachers were informed of each course's release and sent an access code for their students.

Testing of the application took place during March and April 2026. Students were contacted through their teachers. The teachers were tasked with administering an initial diagnostic test to the students and regularly engaging them in active use of the application at least once a week for at least 15 minutes.

After basic data pruning, anonymous access and inactive users with insufficient recorded interaction data were excluded. The resulting dataset included 63,020 interactions among 628 students across 43 classes and 233 tasks. During actual testing, the model's results were not made available to the students at this stage to avoid any potential undue influence on the testing process.

4. Results of the Replay-Based Simulation

The replay-based simulation preserved the original chronological sequence of learner–task interactions, including achieved scores and scaffold use, while recalculating learner and task Elo parameters under iteratively adjusted equilibrium settings. Each replay informed the subsequent parameter adjustment. The aim was not exhaustive optimisation, but identification of a stable and pedagogically interpretable configuration. The goal of the simulation was to:

- analyse the stability of learner and task profiles,
- monitor the impact of scaffold-aware scoring,
- analyse learner growth,
- stabilise task dynamics.

Configurations were evaluated primarily for the absence of divergent behaviour, excessive short-term fluctuations, and unstable task-difficulty dynamics.

4.1 Stable Task Dynamics

The replay-based simulation showed that more stable task dynamics were achieved at target equilibrium values lower than those originally proposed, specifically:

- $target_student_at_equal$ at value 0.8 ($L = D$ theoretically occurs from the student's perspective at a score of 0.8; the student's Elo rating remains unchanged at this value, increases at a score of 1, and decreases at a score < 0.8 according to the parameters of the logistic function,
- $target_task_at_equal$ at value 0.85 ($L = D$ theoretically occurs from the task's perspective at a score of 0.85, when the task's Elo rating remains unchanged; at a score of 1, it decreases slightly, and at a score < 0.85, it increases according to the parameters of the logistic function).

The parameters of the logistics function correspond to these target values:

$$b_L = \ln\frac{0.8}{0.2} \approx 1.386,$$

and

$$b_D = \ln\frac{0.85}{0.15} \approx 1.735.$$

4.2 Relationship Between the Target Equilibrium and the Scoring Policy System

A key finding of the replay-based simulation was the strong link between the target equilibrium parameters and the system's scoring policy. The results also show that the design of pedagogically asymmetric Elo systems cannot be separated from the scoring policy and the method of interpreting the scaffolded solution.

The classical Elo model assumes an equilibrium state at an expected success rate of 50%. This setting is suitable for diagnostic purposes because it maximises the informational value of individual interactions. From a pedagogical perspective, however, it represents a situation in which the learner fails in approximately half of cases, which may not be suitable for long-term learning or maintaining motivation.

In the proposed model, therefore, the learner equilibrium is shifted to 0.80. Given the same learner level and task, the system assumes an approximately 80% probability of a successful solution. The goal of this setting is to create an environment where the learner predominantly experiences success while remaining sufficiently cognitively challenged.

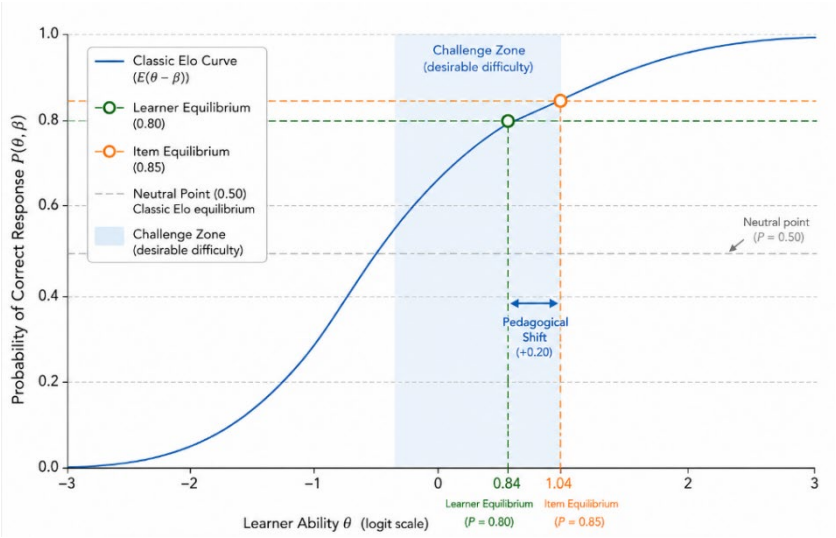


Figure 2: A Shift of Pedagogical Equilibrium

The balance for updating task difficulty is set even more conservatively. Based on simulations, the task equilibrium was set to 0.85. Task difficulty, therefore, stabilises only when students' long-term success rate reaches approximately 85%. This setting limits overly rapid increases in task difficulty while simultaneously supporting the maintenance of the so-called "challenge zone"—an area of productive challenge where tasks are sufficiently demanding to foster skill development but do not lead to systematic failure.

Replay-based simulations have shown that even small changes to these parameters significantly influence the system's long-term dynamics. Shifting the learner equilibrium increases the likelihood of growth in learner profiles, while a higher task equilibrium contributes to the stability of task difficulty. Experimentally, the most balanced configuration was achieved with the settings $target_student_at_equal = 0.80$ and $target_task_at_equal = 0.85$ (Figure 2).

Replay-based simulations revealed a strong interdependence between learner and task dynamics. Even relatively small changes in pedagogical balance parameters significantly influenced the distribution of increases and decreases in learner profiles (Figure 3).

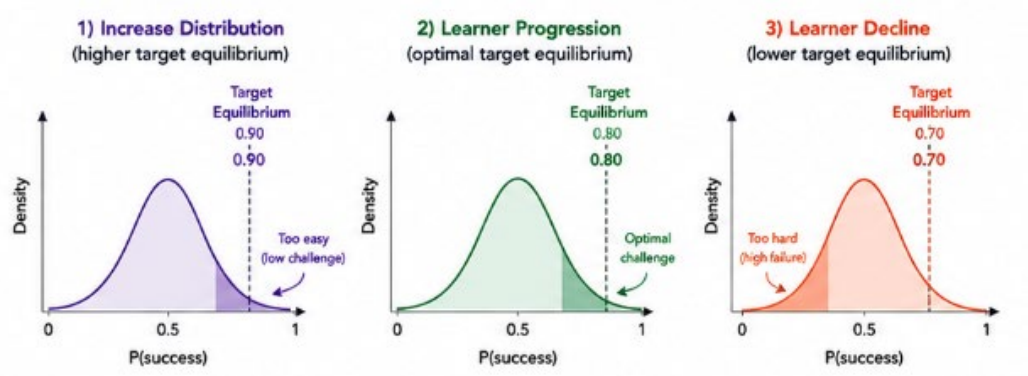


Figure 3: The effect of target equilibrium parameters on learner growth

The most stable configuration was achieved experimentally with:

- $target_student_at_equal = 0.8$,
- $target_task_at_equal = 0.85$.

4.3 Task Convergence Behaviour

To illustrate the model's behaviour, the trajectories of the Elo parameters for individual tasks were analysed within specific components of the competence vector.

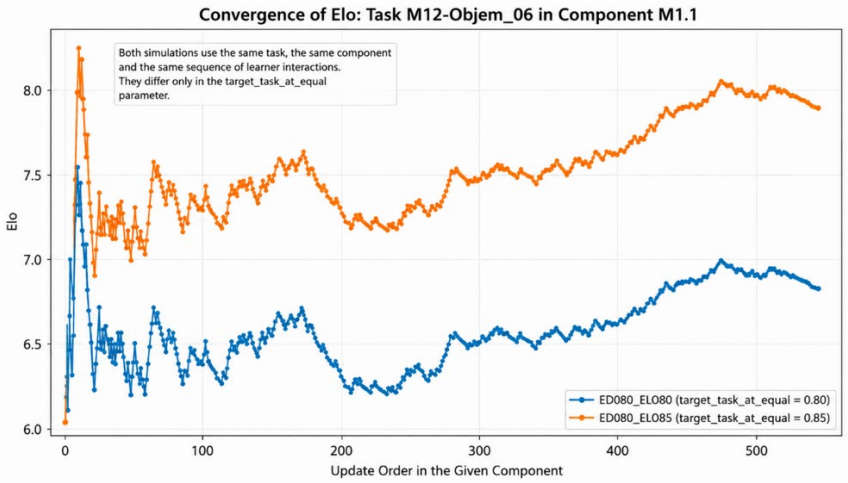


Figure 4: Convergence of task difficulty for task M12-Objem_06 in Component M1.1

Figure 4 shows the evolution of the difficulty parameter for task *M12-Objem_06* in component M1.1 across two model configurations. This task was selected as an illustrative example to demonstrate how the same interaction history produces different difficulty trajectories under different task-equilibrium settings; it was not selected as a statistically representative case.

Both simulations are based on the same task, the same component of the competence vector, and the same historical sequence of learner–task interactions. They differ only in the setting of the *target_task_at_equal* parameter. In both cases, the estimate shows high sensitivity in the initial phase, when only a small number of responses are available. As the number of interactions increases, the oscillations gradually dampen, and the difficulty parameter stabilises.

Importantly, however, the two configurations do not converge to the same value. The ED080_ELO80 configuration stabilises at a lower estimated difficulty, while the ED080_ELO85 configuration converges to a significantly higher value. This shows that the target equilibrium setting not only affects short-term fluctuations in the model but also systematically shapes the resulting estimate of the task's difficulty.

It is also significant that the system does not exhibit long-term uncontrolled drift in any configuration. After enough interactions, changes in difficulty gradually diminish, and the task parameter oscillates within a limited range around a stable value. At the same time, the result confirms that the choice of parameters must be evaluated not only in terms of task stability but also in relation to the dynamics of learner profiles, since task calibration and the growth of learner parameters are recursively linked in the model.

4.4 Learner Convergence Behaviour

In addition to the stability of task parameters, the dynamics of the learner model were also monitored. Figure 5 shows the development of Student 213's Elo parameter in component A3.3 across two model configurations.

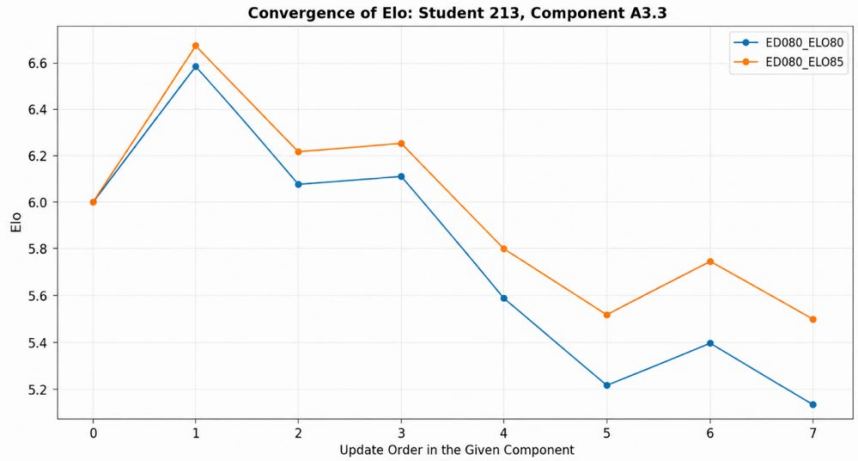


Figure 5: Convergence of learner parameter for student 213 in component A3.3

Student 213 was selected because this learner had a comparatively high number of updates in component A3.3, allowing the trajectory to be illustrated over multiple iterations. The figure is therefore intended as an illustration of model dynamics rather than as evidence based on a representative individual case.

Unlike task parameters, learner parameters exhibit greater short-term variability. This is to be expected, as the number of interactions within the individual components of the competence vector is significantly lower than the number used to estimate task difficulty.

The graph shows that the ED080_ELO85 configuration yields higher learner parameter values in the observed component than the ED080_ELO80 configuration. This difference suggests that the choice of target equilibrium parameters significantly influences not only task stability but also the learner profile's trajectory. In this case, the ED080_ELO85 configuration better aligns with the pedagogical expectation that an active and regularly working student should not be systematically shifted downward by the model.

5. Discussion

The proposed model represents a shift from psychometrically oriented adaptive systems toward pedagogically oriented adaptive learning. The fundamental difference lies in the redefinition of the adaptation goal. While traditional systems primarily optimise the accuracy of ability estimates, the proposed system is designed to support pedagogically appropriate learning conditions through asymmetric learner and task dynamics. A significant contribution is the multidimensional representation of knowledge, which enables more detailed modelling of mathematical competencies. The integration with the Global Proficiency Framework simultaneously increases the model's interpretability and enables diagnostic feedback to both teachers and students.

Replay-based simulations demonstrated the importance of stabilised updates to task difficulty values. Without limiting the update rate, faster growth of learner profiles led to unstable increases in task difficulty. At the same time, however, it turned out that the stability of task parameters alone is not a sufficient criterion for model quality. Configurations that yield more stable estimates of task difficulty may also result in undesirable declines in learner profiles.

A key finding was the strong interdependence between learner and task dynamics. It turned out that even small changes in target equilibrium parameters significantly influence the distribution of learner growth and learner decline. This result confirms the nonlinear nature of recursively defined adaptive systems and shows that parameter settings cannot be evaluated in isolation.

The system’s scoring policy plays a crucial role in this process. The original assumption pointed toward higher target success rates; however, the inclusion of hints and the use of support-adjusted scoring altered the effective distribution of achieved scores. A successful solution with a hint does not carry the same weight as an independent solution, so it was necessary to adjust the target values for learner and task equilibrium to 0.80 and 0.85, respectively.

The results also suggest that adjusting task difficulty should not be the system’s sole response to students’ repeated difficulties. If a particular task yields lower-than-expected success rates, this does not necessarily mean it is poorly calibrated. It may also mean that students need better preparation, more appropriate scaffolding, additional explanations, or a different sequence of preceding activities. Future development of the system should therefore focus on identifying pedagogically relevant interactions and designing supportive interventions that increase the likelihood of successfully mastering a given competency.

The pilot implementation also revealed several limitations of the system. The competency taxonomy is still being refined, and AI-assisted task classification may contain minor inconsistencies. Another limitation is the relatively low number of interactions in some components of the learner vector, which may increase the short-term variability of learner parameters. At the same time, the study did not directly measure learning gains, but focused primarily on the stability and pedagogical interpretability of the proposed model. The identified equilibrium settings should therefore be regarded as promising initial configurations rather than as optimal values for learning effectiveness.

6. Conclusion

This article presented a pedagogically asymmetric parameterisation of a multidimensional Elo system designed for adaptive mathematics education. The proposed approach shifts the use of the Elo model from a purely

diagnostic assessment of abilities toward supporting the learning process through pedagogically differentiated learner and task dynamics. A pilot implementation in the Edu-AI system and replay-based simulations utilising more than 63,000 historical interactions demonstrated the potential of this approach and showed that target-equilibrium settings substantially influence the long-term dynamics of learner growth and task stability. The scoring policy, which reflects scaffolded task-solving, also plays a significant role here. Therefore, the setting of target values cannot be separated from the way the system evaluates independent solutions, solutions with hints, and unsuccessful attempts.

The most balanced configuration observed in the replay-based simulation used the equilibrium settings 0.80 for learner updates and 0.85 for task updates. These values should be regarded as promising initial settings rather than as optimal parameters for learning effectiveness. Future research will focus on further refining the competency taxonomy, prospectively testing and adjusting equilibrium parameters, identifying pedagogically relevant interactions, and analysing the system's impact on students' educational outcomes.

Acknowledgements

This paper was supported by the project TAČR N. TQ23000334 – AI-based Adaptive Learning Platform.

Ethics Statement

Participation occurred only in classes enrolled in the pilot through their teachers, and the required informed consent was obtained before participation. The analytical dataset contained no personally identifiable information and was processed in anonymised form.

AI Declaration

Generative AI tools were used in preparing this article for graphic design, language support, and text organisation. The authors of this article developed all technical concepts, methodologies, and interpretations.

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