Cognitive Barriers in Solving Context-Based Geometry Tasks in an LMS Moodle Environment

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Abstract: Context-based tasks provide an important means of connecting geometric knowledge with real-world situations. Successfully solving such tasks, however, requires more than knowledge of geometric concepts and relationships. Students need to understand the given situation, translate it into an appropriate geometric model, identify the relevant geometric relationships, and subsequently interpret the obtained solution within the original context. Each stage of this process may give rise to specific difficulties and cognitive barriers, which can be reflected in incorrect or incomplete solutions. This process can be viewed within the mathematical modelling cycle described by Blum and Leiß (2007). This study focuses on the analysis of open-ended responses produced by pre-service mathematics teachers when solving context-based geometry tasks in the Synthetic Geometry I course. One of the key topics addressed in the course is loci of points satisfying given conditions. As part of the course, students completed an e-test in the Moodle learning management system consisting of seven open-ended context-based tasks. Solving these tasks required students to translate real-world spatial situations into two-dimensional geometric models, identify the corresponding loci of points, and interpret their solutions in relation to the original problem context. Open-ended responses collected during two implementations of the course were analysed using qualitative content analysis with inductively developed categories. Particular attention was paid to incorrect and incomplete solutions in order to identify recurring error patterns and, on this basis, to characterise potential cognitive barriers arising at different stages of the problem-solving process. The analysis focuses particularly on difficulties related to understanding the problem context, constructing an appropriate geometric model, identifying the relevant locus of points, and interpreting the geometric solution within the original situation. The study highlights the potential of analysing incorrect responses as a means of gaining deeper insight into the geometric reasoning of pre-service mathematics teachers. Identifying cognitive barriers may also contribute to the more targeted design of instruction and e-assessment tasks that support not only the acquisition of geometric knowledge but also the development of students' ability to mathematically model and interpret real-world situations.

Keywords: Open-Ended E-assessment, Mathematical Modelling, Geometry Education, Cognitive Barriers, Pre-service Mathematics Teachers.

1. Introduction

Against this background, the present study analyses open-ended responses collected through a Moodle-based e-test in a Synthetic Geometry course for pre-service mathematics teachers. The test consisted of seven context-based tasks concerning loci of points satisfying given conditions. Previous research has addressed digital assessment in mathematics and the formative potential of Moodle-based assessment (Blanco and Ginovart, 2012; Drijvers, 2018; Weigand, Trgalova and Tabach, 2024). However, less attention has been paid to how open-ended responses collected through LMS-based e-assessment can be systematically analysed to distinguish qualitatively different patterns of difficulty in context-based geometry tasks. Rather than focusing solely on achievement, the present study examines incorrect and incomplete responses to identify recurring diagnostic patterns and potential cognitive barriers. The modelling cycle of Blum and Leiß (2007) is subsequently used as an interpretive lens for considering where difficulties may arise when students translate contextual situations into geometric representations. The study therefore contributes to research on e-assessment by demonstrating how open-ended response data collected in Moodle can provide diagnostic information beyond binary scoring, while also offering a geometry-specific framework for interpreting recurring patterns of difficulty.

2. Theoretical Background

Understanding students' difficulties in context-based geometry tasks requires several interconnected perspectives. Digital assessment provides an environment for systematically collecting students' responses, while mathematical modelling offers a framework for understanding the transition between real-world situations and mathematical representations. The analysis of incorrect and incomplete responses can further reveal recurring patterns of difficulty. The following sections therefore address e-assessment in mathematics

and LMS Moodle, mathematical modelling in context-based geometry tasks, and errors and potential cognitive barriers in mathematical problem solving.

2.1 E-Assessment in Mathematics and LMS Moodle

Digital assessment has become an established component of higher education, providing opportunities not only for efficient test administration but also for collecting information about students' learning and performance. Online assessment can support formative processes through feedback and the systematic collection of learner data that may inform subsequent instructional decisions (Heil and Ifenthaler, 2023). In mathematics education, however, digital assessment presents specific challenges because mathematical competence cannot always be adequately represented by selecting a predefined answer or entering only a final numerical result.

Mathematical problem solving may involve multiple representations, strategies, and intermediate steps. Drijvers (2018) therefore emphasises that digital mathematics assessment should provide sufficiently rich opportunities for students to engage in mathematical activity and notes the limitations of assessment focused primarily on final answers and automated scoring. This issue is particularly relevant for open-ended tasks, where an incorrect or incomplete response may contain information about students' reasoning that would remain hidden in a simple correct/incorrect score.

Learning management systems such as Moodle provide an accessible environment for integrating e-assessment into university mathematics courses. Moodle supports different question formats, feedback, repeated testing, and the systematic storage of individual responses. Blanco and Ginovart (2012) demonstrated the potential of Moodle quizzes for formative e-assessment in university mathematics and showed that assessment data can be used not only to evaluate students' performance but also to inform subsequent teaching and assessment.

In the present study, Moodle was used to administer an e-test containing open-ended context-based geometry tasks and to systematically collect students' responses. These responses provide the data for a qualitative analysis of incorrect and incomplete solutions. Thus, Moodle-based e-assessment is considered not only as a means of administering and scoring tasks but also as an environment that enables the collection of response data for examining recurring patterns of difficulty in students' geometric problem solving. Its diagnostic potential therefore lies in preserving open-ended response data for subsequent qualitative analysis rather than in automated scoring itself.

2.2 Context-Based Geometry Tasks and Mathematical Modelling

Context-based mathematical tasks require students to connect mathematical knowledge with situations outside purely mathematical contexts. Their solution involves identifying mathematically relevant aspects of a situation, representing them appropriately, applying mathematical concepts and methods, and interpreting the obtained result in relation to the original context. Mathematical modelling provides a theoretical framework for describing these transitions between reality and mathematics (Blum and Niss, 1991; Kaiser and Sriraman, 2006; Stillman, 2019).

One influential framework is the modelling cycle proposed by Blum and Leiß (2007), which distinguishes between the real and mathematical worlds and conceptualises modelling as a sequence of interconnected processes (Figure 1). Starting from a real situation, the solver constructs a situation model, which is subsequently simplified and structured into a real model. Through mathematisation, this is transformed into a mathematical model, within which mathematical reasoning produces mathematical results. These results are then interpreted as real results and validated against the original situation (Blum and Leiß, 2007).

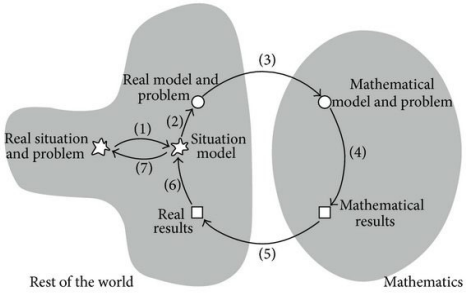


Figure 1: Mathematical modelling cycle according to Blum and Leiß (2007)

The modelling cycle should not be interpreted as a rigid linear sequence. Students may move between stages, return to previous phases, or follow individual modelling routes during problem solving (Borromeo Ferri, 2006; Greefrath and Vorhölter, 2016). Difficulties encountered in modelling tasks therefore cannot automatically be attributed to insufficient mathematical knowledge, as they may arise at different points in the modelling process.

This perspective is particularly relevant to context-based geometry tasks, where students need to translate real or spatial situations into appropriate geometric representations. In tasks involving loci of points satisfying given conditions, conditions expressed in contextual terms must be connected with corresponding geometric properties, and the resulting locus must subsequently be interpreted within the original situation. Research on geometric modelling indicates that mathematisation can be challenging when information derived from real objects or their representations needs to be transformed into mathematical models (Jablonski, 2024).

In the present study, the modelling cycle is used as a theoretical lens rather than a predetermined coding framework. Categories of incorrect and incomplete responses are developed inductively from the data and subsequently considered in relation to transitions within the modelling process. This approach enables recurring response patterns to be examined as possible indicators of difficulties or potential cognitive barriers without assuming that a particular written error directly reflects a specific cognitive process.

2.3 Errors and Cognitive Barriers in Mathematical Problem Solving

Errors can provide valuable information about students' mathematical understanding and reasoning rather than serving solely as indicators of failure. Borasi (1994) described mathematical errors as potential "springboards for inquiry", highlighting their value for examining students' thinking. Error analysis can therefore contribute to identifying recurring areas of difficulty that may inform both research and instructional decisions.

An important distinction must, however, be made between an observable error and its underlying cause. An incorrect response provides evidence that a difficulty occurred but does not necessarily reveal the cognitive process that produced it. Similar errors may originate from different sources, including conceptual misunderstanding, inappropriate strategy selection, difficulties in interpreting the task, or procedural problems. Consequently, recurring patterns across students' responses are more informative than treating individual errors as direct evidence of particular misconceptions or cognitive processes (Radatz, 1980; Borasi, 1994).

This distinction is particularly relevant in geometry, where problem solving involves conceptual knowledge, visualisation, representation, and mathematical reasoning. Research involving pre-service mathematics teachers has identified difficulties related to applying geometric concepts, visualising geometric objects, understanding problem situations, and selecting appropriate principles or definitions (Noto, Priatna and Dahlan, 2019). An incorrect geometric response should therefore not automatically be interpreted as evidence of insufficient geometric knowledge.

In the present study, the term potential cognitive barriers is used cautiously to describe difficulties that may be reflected in recurring patterns of incorrect or incomplete responses. Categories are developed inductively from students' written responses rather than assigned according to presumed cognitive processes. These patterns are subsequently considered in relation to the mathematical modelling framework described above. This approach allows the analysis to explore the diagnostic information contained in students' responses while avoiding direct inference about cognitive processes that cannot be established from final written responses alone. Together, these perspectives suggest a need to examine how open-ended e-assessment responses can provide diagnostic information about qualitatively different difficulties in context-based geometry tasks.

2.4 Research Questions

Building on the theoretical considerations outlined above, the study examines the diagnostic potential of open-ended e-assessment in context-based geometry tasks. Particular attention is paid to the recurring characteristics of students' responses, their possible interpretation through the mathematical modelling cycle, and the diagnostic information that can be obtained beyond binary correct/incorrect scoring. The study addresses the following research questions:

> RQ1: What recurring diagnostic patterns emerge from pre-service mathematics teachers' open responses to context-based geometry tasks in a Moodle-based e-assessment?

RQ2: How can these patterns be interpreted in relation to the stages of the mathematical modelling cycle proposed by Blum and Leiß (2007)?

RQ3: What diagnostic potential do open-ended responses in Moodle-based e-assessment offer beyond the distinction between correct and incorrect answers?

3. Methodology

3.1 Research Context and Participants

The study was conducted within the *Synthetic Geometry I* course for prospective mathematics teachers at the Faculty of Education, University of Ostrava. Loci defined by geometric properties constitute one of the key topics addressed in the course. As part of the course, students completed an e-test administered through the LMS Moodle. The e-test included seven open-ended context-based geometry tasks designed around real-world situations that could be represented and solved using concepts related to geometric loci.

The dataset comprised anonymised open-ended responses from 33 prospective mathematics teachers, collected during two implementations of the course. The first cohort consisted of 16 students and the second of 17 students, yielding a total of 231 responses across the seven tasks. Responses from the two course implementations were analysed as a single dataset, as the purpose of the study was to identify recurring diagnostic patterns rather than to compare the two cohorts. The analysis focused on the information contained in students' written responses rather than solely on whether the responses corresponded to the expected solutions.

3.2 Design of the E-Assessment Tasks

The e-test consisted of seven open-ended tasks focusing on loci defined by geometric properties. Each task presented a real-world context requiring students to translate a spatial situation into plane geometry, identify the locus corresponding to the specified conditions, and interpret the solution in relation to the original context. The tasks therefore reflected the general logic of mathematical modelling described by Blum and Leiß (2007). An overview of the task contexts and expected geometric solutions is provided in Table 1.

Table 1: Overview of the e-assessment tasks and expected geometric solutions

Task	Context	Expected Geometric Solution
T1	Path 10 m from a straight riverbank	Two lines parallel to the riverbank at a distance of 10 m
T2	Area reachable by a goat tied to a stake	Disk centred at K with radius 1 m
T3	Ship remaining 30 km from a lighthouse	Circle centred at the lighthouse with radius 30 km
T4	Cycle path midway between two parallel roads	Line parallel to and equidistant from both roads
T5	Hydrants equidistant from two intersecting streets	Angle bisector of the 60° angle formed by the two streets
T6	Zone equidistant from two sound towers A and B	Perpendicular bisector of segment AB
T7	Watering point equidistant from three barns A, B and C	Circumcentre of triangle ABC (intersection of the perpendicular bisectors of its sides)

The open-ended format required students to formulate their solutions independently rather than select from predefined alternatives. This enabled the responses to preserve evidence of incomplete representations, substitutions between geometric concepts, terminological or formalisation difficulties, procedural reasoning, and internal inconsistencies that could remain obscured by binary correct/incorrect scoring.

3.3 Data Analysis and Development of the Diagnostic Coding Framework

The 231 anonymised responses were analysed using qualitative content analysis with inductive category development. Both authors independently analysed the complete dataset. Observable characteristics of the responses were coded without attributing them to presumed cognitive processes. The independently generated codes were compared, and disagreements or ambiguous cases were discussed until consensus was reached. No

inter-rater agreement statistic was calculated, as the independent analyses were used to support qualitative comparison and consensus development rather than to assess coder reliability statistically. Analytical trustworthiness was supported through independent analysis of the complete dataset by two authors, systematic comparison of the resulting codes, and consensus discussion of disagreements and ambiguous cases. Recurring codes were subsequently grouped into seven higher-order diagnostic patterns. The complete framework is presented in Table 2.

Table 2: Diagnostic coding framework

Higher-Order Pattern	Open Code	Definition and Inclusion/Exclusion Criteria	Representative Example
Successful geometric representation	C1 Explicit correct geometric identification	The expected geometric object or locus is correctly and explicitly identified using appropriate geometric terminology. Includes responses that specify the essential conditions required by the task. Excludes incomplete representations and responses in which the relationship is described correctly but the standard geometric term is not used.	"Circle centred at the lighthouse with radius 30 km."
Successful geometric representation	C2 Correct descriptive identification without standard term	The expected geometric relationship or locus is correctly described, but the conventional geometric term is not explicitly used. Includes responses demonstrating the relevant relationship through an accurate verbal description. Excludes terminologically inaccurate expressions and incomplete representations.	Correctly describes the set of points equidistant from A and B without naming the perpendicular bisector.
Incomplete geometric representation	C3 Incomplete / under-specified locus or object	A relevant component of the expected geometric representation is identified, but the complete locus or all required conditions are not represented. Includes omission of a necessary component, distance, radius, or other defining condition. Excludes responses based on a different geometric concept.	"Rovnoběžka s břehem řeky ve vzdálenosti 10 m." — only one of the two required parallel lines is identified.
Conceptual geometric substitution	C4 Substitution by a different geometric object/locus	The expected geometric object or locus is replaced by another geometric object or locus. Excludes responses that identify the correct object but omit only a defining condition.	A different locus is identified instead of the expected one.
Conceptual geometric substitution	C5 Boundary–region distinction not maintained	The response does not distinguish correctly between the boundary of a geometric set and the entire region satisfying the contextual condition.	*Kružnice* [circle] instead of *kruh* [disk] for the area reachable by the goat.
Conceptual geometric substitution	C6 Substitution among notable triangle points	The required notable point of a triangle is replaced by another notable point. Includes substitutions among geometrically related triangle centres.	*Těžiště* [centroid] instead of the circumcentre.
Terminological/formalisation issue	C7 Terminological imprecision / non-standard formalisation	The relevant geometric relationship is recognisable, but it is expressed using inaccurate, non-standard, or insufficiently formal geometric terminology. Excludes responses in which a genuinely different geometric concept is substituted.	An appropriate geometric relationship expressed using non-standard terminology.
Observable procedural reasoning	C8 Construction-oriented expression of the solution	The response contains an observable construction or procedural route used to obtain the geometric solution rather than merely identifying the resulting locus.	"uděláme kolmici k úsečce AB přesně ve středu" [we construct a perpendicular to segment AB exactly at its midpoint].
Mixed/inconsistent or ancillary error	C9 Internally inconsistent / mixed conceptual response	The response contains mutually incompatible geometric statements or combines an appropriate idea with a contradictory geometric identification.	A response correctly referring to the circumcentre and subsequently

Higher-Order Pattern	Open Code	Definition and Inclusion/Exclusion Criteria	Representative Example
			identifying the same point as the centroid.
Mixed/inconsistent or ancillary error	C10 Ancillary numerical / unit error	The principal geometric representation is appropriate but is accompanied by a secondary numerical, measurement, or unit-related error. Excludes errors that change the underlying geometric concept itself.	Correct locus accompanied by an incorrect numerical value or unit.
Non-interpretable response	C11 No response / insufficiently interpretable response	The response is absent, excessively vague, or contains insufficient information to support a reliable diagnostic interpretation. No specific conceptual difficulty is inferred from such a response.	Blank response or a statement too limited to determine the intended geometric representation.

3.4 Post-Hoc Interpretation Through the Mathematical Modelling Cycle

After the diagnostic coding framework had been developed inductively, the identified response patterns were considered in relation to the mathematical modelling cycle proposed by Blum and Leiß (2007). The modelling cycle therefore served as a post-hoc interpretive lens rather than an a priori coding framework**.**

The analysis considered how the identified patterns could plausibly relate to processes involved in translating a real-world situation into a mathematical representation, working within the mathematical model, and relating mathematical results to the original context. These associations were interpreted cautiously because the dataset consisted of final written responses rather than process-oriented data. Individual responses were therefore not treated as direct evidence of the cognitive stage at which a difficulty occurred.

Finally, the identified patterns were considered from an e-assessment perspective, focusing on the diagnostic information provided by open-ended responses beyond binary correct/incorrect scoring.

4. Results

4.1 Recurring Diagnostic Patterns in Students' Open Responses

The analysis of 231 open responses across the seven context-based geometry tasks identified 11 open codes, which were organised into seven higher-order diagnostic patterns. As individual responses could display more than one analytically relevant characteristic, the diagnostic patterns were not treated as mutually exclusive categories. The overall distribution of the higher-order diagnostic patterns is presented in Table 3.

Table 3: Distribution of Higher-Order Diagnostic Patterns Identified in Students' Open Responses

Diagnostic Pattern	n	% of Responses	Diagnostic Characteristic
Successful geometric representation	126	54.5	Expected geometric relationship identified explicitly or descriptively
Incomplete geometric representation	36	15.6	Relevant representation present but incomplete or under-specified
Conceptual geometric substitution	34	14.7	Another geometric object, locus, region/boundary concept, or notable point substituted
Terminological/formalisation issue	8	3.5	Geometric relationship recognisable but expressed imprecisely
Observable procedural reasoning	2	0.9	Construction or procedural route visible in the response
Mixed/inconsistent or ancillary error	5	2.2	Contradictory concepts or an additional numerical/unit error
Non-interpretable response	44	19.0	Insufficient evidence for reliable diagnostic interpretation

The most frequently identified pattern was successful geometric representation, observed in 126 responses (54.5%). This pattern comprised both explicit identification of the expected geometric concept (117 responses; 50.6%) and correct descriptive identification without the use of the standard geometric term (9 responses;

3.9%). The latter distinction was diagnostically relevant because it indicated that an appropriate geometric relationship could be recognised even when formal terminology was not used.

The second major pattern was incomplete geometric representation, identified in 36 responses (15.6%). In these cases, the response contained a relevant component of the expected geometric model but did not fully represent the locus or all conditions specified in the task. A characteristic example occurred in the riverbank task, in which a student responded: *"Rovnoběžka s břehem řeky ve vzdálenosti 10 m."* [A line parallel to the riverbank at a distance of 10 metres.]. The response identifies the relevant parallel direction and distance but specifies only one of the two parallel lines forming the complete locus. Thus, the response demonstrates partial geometric understanding that would be obscured if it were recorded only as incorrect.

A comparable proportion of responses displayed conceptual geometric substitution (34 responses; 14.7%). This higher-order pattern included substitution of the expected locus by another geometric object (12 responses; 5.2%), failure to maintain the distinction between a boundary and a region (14 responses; 6.1%), and substitution among notable points of a triangle (10 responses; 4.3%). For example, in a task asking for the entire region accessible to a goat tied to a fixed point, some students identified a *kružnice* [circle] rather than a *kruh* [disk]. In another task, *těžiště* [centroid] was given instead of the centre of the circumscribed circle. Such responses did not merely represent an absence of knowledge; they revealed which alternative geometric construct had been activated.

Terminological or formalisation issues were identified in eight responses (3.5%). These responses indicated an appropriate or recognisable geometric relationship but expressed it using imprecise or non-standard terminology. This pattern was therefore distinguished from conceptual substitution: the difficulty concerned the formal expression of the geometric idea rather than necessarily the underlying identification of the relevant relationship.

Less frequent but diagnostically informative patterns were also identified. Observable procedural reasoning occurred in two responses (0.9%), in which students described a construction procedure rather than merely naming the resulting locus. Mixed or internally inconsistent responses and ancillary errors occurred in five responses (2.2%). These included responses combining mutually incompatible geometric concepts or an otherwise appropriate geometric solution with an additional numerical or unit-related error.

Finally, 44 responses (19.0%) were classified as non-interpretable because they were absent or did not contain sufficient information to support a reliable diagnostic interpretation. These responses were retained as a separate pattern rather than being assigned to a presumed conceptual difficulty for which the written data provided no evidence.

Table 4: Distribution of Higher-Order Diagnostic Patterns across the Seven E-assessment Tasks

Task	Successful representation	Incomplete representation	Conceptual substitution	Terminological/ formalisation issue	Procedural reasoning	Mixed/inconsistent or ancillary error	Non-interpretable
T1	2	24	6	0	0	0	1
T2	20	0	13	0	0	3	0
T3	27	10	4	0	0	1	2
T4	22	0	0	8	0	0	10
T5	16	0	0	0	0	0	17
T6	30	1	1	0	2	0	1
T7	9	1	10	0	0	1	13

Note: Diagnostic patterns were not mutually exclusive; therefore, the frequencies within individual tasks may exceed the number of responses (n = 33 per task).

The distribution of diagnostic patterns varied across the seven tasks (Table 4). Incomplete geometric representation was particularly prominent in Task 1 (24/33), whereas conceptual geometric substitution was frequent in Task 2 (13/33) and Task 7 (10/33). Task 6 showed the highest frequency of successful geometric representations (30/33), while Task 7 also generated a comparatively high number of non-interpretable responses (13/33). Terminological or formalisation issues were concentrated in Task 4 (8/33), and observable procedural reasoning occurred only in Task 6 (2/33). These task-specific differences indicate that the diagnostic patterns were associated with the particular geometric distinctions elicited by individual contextual situations rather than representing a uniform type of difficulty across the e-test.

4.2 Interpretation of the Diagnostic Patterns Through Mathematical Modelling

The diagnostic patterns were subsequently interpreted in relation to the mathematical modelling cycle of Blum and Leiß (2007), using the cycle as a post-hoc interpretive framework rather than as a basis for the original coding.

Incomplete geometric representations may be consistent with incomplete mathematisation. For example, identifying one parallel line at the specified distance from a riverbank demonstrates recognition of the relevant relationship while failing to represent the complete locus.

Conceptual geometric substitutions may reflect difficulty in constructing the mathematical model or working mathematically with the concepts involved. For example, distinguishing between a circle and a disk requires determining whether the contextual condition specifies a boundary or an entire region. Similarly, substituting the centroid for the circumcentre indicates difficulty in differentiating between related geometric constructs.

Responses displaying procedural reasoning contained evidence of working mathematically within the geometric model, whereas terminological or formalisation difficulties could not be reliably associated with a specific modelling process.

Overall, the modelling cycle provides a framework for considering processes potentially implicated by the observed patterns. Final written responses, however, do not allow the cognitive processes underlying individual difficulties to be reliably reconstructed.

4.3 Diagnostic Information Beyond Correct/Incorrect Scoring

The analysis demonstrated that open-ended responses provided diagnostic information beyond binary correct/incorrect scoring. Responses receiving the same incorrect outcome differed in their mathematical structure, including incomplete representations, conceptual substitutions, procedural reasoning, terminological difficulties, and internal inconsistencies.

In the Moodle-based e-test, open-ended responses preserved students' individual formulations, allowing these qualitatively different patterns to be distinguished. Their diagnostic value therefore lies not in automatically identifying students' cognitive difficulties, but in providing richer evidence for subsequent qualitative analysis and feedback.

5. Discussion

5.1 From Correctness to Diagnostic Interpretation in Open-Ended E-Assessment

Responses receiving the same incorrect score may reflect qualitatively different forms of mathematical understanding and therefore require different instructional interpretations. An incomplete geometric representation, conceptual substitution, and terminological imprecision represent different diagnostic situations despite potentially receiving the same incorrect score.

The findings therefore support the use of open-ended e-assessment as a source of diagnostic evidence that may inform more differentiated feedback and subsequent instructional decisions.

5.2 Context-Based Geometry Tasks and Mathematical Modelling

The identified patterns suggest that difficulties in context-based geometry tasks cannot be reduced to insufficient knowledge of geometric concepts. Students must translate contextual situations into mathematical representations, creating several points at which difficulties may emerge. This supports previous findings that mathematisation can be particularly demanding when information from real or represented situations must be transformed into an appropriate mathematical model (Jablonski, 2024).

The distinction between incomplete representations and conceptual substitutions further shows that similar incorrect outcomes may reflect qualitatively different difficulties. These patterns should, however, be interpreted as potential indicators of cognitive barriers rather than direct evidence of specific cognitive processes.

5.3 Diagnostic Affordances of Open-Response E-Assessment

Open-ended responses can provide a richer basis for feedback by distinguishing between qualitatively different difficulties that would remain indistinguishable under binary scoring.

Such distinctions may support more targeted feedback. For example, an incomplete representation may require attention to omitted conditions, whereas a conceptual substitution may call for clarification of relationships between geometric concepts. Terminological imprecision may instead require support in mathematical communication. This aligns with the formative potential of digital assessment, in which response data can inform subsequent instructional decisions (Blanco and Ginovart, 2012; Heil and Ifenthaler, 2023).

However, this diagnostic value does not arise automatically from the digital environment. Moodle facilitates the collection and preservation of open-ended responses, but their interpretation requires an appropriate analytical framework and professional judgement. Moodle-based e-assessment can therefore support differentiated feedback when responses are treated not only as objects of scoring but also as diagnostic evidence.

5.4 Implications for Prospective Mathematics Teacher Education

The findings are particularly relevant to pre-service mathematics teacher education. Prospective teachers need not only to solve geometric problems correctly but also to recognise different forms of difficulty in students' solutions. Analysing open-ended responses can highlight distinctions between partial understanding, conceptual substitution, and imprecise mathematical communication that may be obscured by correctness-based assessment.

Such analysis may support the development of prospective teachers' diagnostic competence by encouraging them to consider what mathematical understanding is visible in a response and what instructional support may be appropriate. Open-ended e-assessment can thus serve not only as an assessment tool but also as a resource for interpreting students' mathematical thinking.

5.5 Limitations and Directions for Further Research

The study is limited by its context-specific sample of 33 pre-service mathematics teachers from a single university course and is not intended to provide statistically generalisable conclusions. Moreover, final written responses do not allow the cognitive processes underlying particular errors to be reconstructed; the identified patterns should therefore be interpreted as potential indicators of cognitive barriers rather than direct evidence of specific cognitive processes.

Future research could combine open-ended e-assessment with process-oriented methods, such as think-aloud protocols, interviews, or observations, to investigate how particular response patterns emerge. Applying the diagnostic framework to larger samples, different geometric topics, or other mathematical domains could further examine its transferability and refine the identified categories.

6. Conclusion

This study examined the diagnostic potential of open-ended Moodle-based e-assessment through responses to context-based geometry tasks completed by pre-service mathematics teachers. The findings showed that responses receiving the same broad correct/incorrect judgement may reflect qualitatively different forms of mathematical understanding, including incomplete geometric representations, conceptual substitutions, terminological or formalisation difficulties, and internally inconsistent responses.

The identified diagnostic patterns were subsequently interpreted through the mathematical modelling cycle of Blum and Leiß (2007). This interpretation highlighted possible difficulties in translating contextual situations into appropriate geometric representations while also demonstrating the methodological limits of drawing conclusions about underlying cognitive processes from final written responses. The modelling cycle was therefore used as a post-hoc interpretive lens rather than as a framework for reconstructing students' cognitive processes.

Overall, the study demonstrates that the value of open-ended Moodle-based e-assessment lies in preserving diagnostically rich response data that would be obscured by binary scoring alone. Such responses can provide a basis for more differentiated analysis and feedback, while the diagnostic framework developed in this study

offers a systematic approach to distinguishing qualitatively different characteristics of students' geometric responses.

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Ethics Declaration

The data analysed in this study were collected as part of regular teaching and assessment activities in the Synthetic Geometry I course. The dataset consisted exclusively of students' written responses to the Moodle-based e-test. Prior to the research analysis, all responses were anonymised and any personally identifiable information was removed. The research analysis was conducted independently of the students' course assessment and had no impact on their grades or academic outcomes.

AI Declaration

Generative AI tools were used to support language editing, improve the clarity and readability of the manuscript, and assist with the organisation and refinement of the text. The authors retained full responsibility for the research design, data analysis, interpretation of the results, and the final content of the manuscript.

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