

From Research to Prototypes: Developing a Digital Game to Foster Fraction Equivalence

Georgios Thoma¹, Julia Bahnmueller^{1,2}, Manuel Ninaus^{2,3} and Korbinian Moeller^{1,2,4}

¹Department of Mathematics Education, Loughborough University, UK

²LEAD Graduate School & Research Network, University of Tübingen, Germany

³Institute of Psychology, University of Graz, Austria

⁴Leibniz-Institut fuer Wissensmedien, Tübingen, Germany

g.thoma@lboro.ac.uk

j.bahnmueller@lboro.ac.uk

manuel.ninaus@uni-graz.at

k.moeller@lboro.ac.uk

Abstract: Fraction understanding is a significant predictor of future math achievement. In addition to more traditional instruction, digital game-based approaches have been found successful in facilitating conceptual understanding of fractions (including aspects such as part-whole relations represented as pie-charts, or measurement, i.e., magnitude in terms of number lines). Nevertheless, the number of existing games that address the aspect of fraction equivalence is limited and if so, these typically replicate common paper-and-pencil tasks like identifying and matching equivalent fractions. This, however, hardly takes advantage of the medium's potential and affordances nor strives for intrinsic integration of the learning content with game mechanics. Seeking to overcome this shortcoming, the present article describes the development of a digital game fostering fraction equivalence. To do so, among others, we developed visual animations illustrating the transitions from circular part-whole representations (pie-charts) to magnitudes on a number line—drawing on previous research and games. In particular, a wheel is presented, whose circumference matches the length of the number line which it rolls along. Players can manipulate the number of spikes on the wheel reflecting the number of parts of the whole (e.g., 2 spikes halve the circle, while 4 cut it into quarters etc.). By rolling the wheel, the spikes puncture the number line thus segmenting it into equal parts, essentially transforming the aspects from part-whole relations to measurement (magnitude) on the number line. Additionally, as punctures of 2 and 4 spikes will coincide at the $\frac{1}{2}$ - $\frac{2}{4}$ location of a 0-1 number line, players can find equivalent fractions. The game under development will be evaluated with late primary/early secondary school students to test its game design, affordances, and possible impact on fraction understanding.

Keywords: Fraction, Equivalence, Development, Education

1. Introduction

Serious Games have been used successfully to assist in teaching, assessment, and learning of mathematics (Pan et al, 2022). Next to practicalities regarding the approaches of serious games' implementation in various settings (home, school, after-school clubs etc.), other important aspects to consider are the mathematical richness of such games, the informed usage of game elements, as well as their appeal to players (Ke, 2008). Math games/apps range from those providing drill-and-practice worksheet-like tasks to those following more common commercial gameplay—with games facilitating fraction learning being no different (Gresalfi et al, 2018). However, they all face the same challenge: on the one hand the need for math games to be built around, and reflect, content of a standard curriculum; on the other, making use and taking advantage of engaging game elements to make the game enjoyable. This important but difficult balance between mathematical content and game elements, has been highlighted in the mathematics education literature (Devlin, 2011), yet is still being explored (e.g., Kiili et al, 2019). While there are practices and frameworks attempting to provide guidance in this regard (e.g., Van Eck et al, 2017), the creation of a generally applicable template for designing a game that is both educationally effective and appealing to learners/players is impossible. Thus, it is important to consider existing math games to identify their advantages and/or possible points for improvement. As such, this article begins with a brief review of fractions as the mathematical content of interest and game elements that foster fraction understanding. Following, a new conceptual idea is introduced for a unique game mechanic to better integrate fraction concepts of part-whole relations (in a pie-chart) and magnitude (along a number line) and show possible facilitation of fraction equivalence. Finally, we describe a prototypical implementation of the new mechanic in a fraction learning game.

Literature review

1.1 Fractions

Fraction understanding is a significant predictor of life prospects more general, and especially future mathematical achievement (Siegler et al, 2012). Simultaneously, fractions are a topic which both students and adults experience considerable difficulties with (DeWolf and Vosniadou, 2015). One origin of these difficulties may be the different—and compared to whole numbers, more complex—representations and concepts of fractions (such as: part-whole relations, magnitude, equivalence, etc.). Area models and pie-charts are commonly used to introduce the concept of part-whole relations to learners (Fazio and Siegler, 2011) often using circular pie-charts, and pizza examples. Importantly, however, it was found that this may lead to potential misconceptions on fraction magnitudes when students only think of a fraction as the shaded part of a whole—such as a pizza slice—and based on this reject $4/3$ as a fraction, because of their belief that it is impossible to give four parts of an object that is divided into three parts (cf. Fuchs et al, 2017). On the other hand, number line representations were found to be a more effective approach when it comes to conveying the concept of fraction magnitude, with several benefits, including introducing rational numbers by expanding the metaphor of the number line beyond natural numbers (Sidney et al, 2019). Regarding fraction equivalence, emphasis typically is on identifying and pairing different symbolic representations of fractions—pairing $1/2$ with 50%, with 0.5, or with other representations (e.g., Figure 1). Interestingly, pie-charts and number lines can also be used to illustrate fraction equivalence, as equivalent fractions have identical shaded areas or are located at the very same spatial location on the number line. Particularly, segmentations of pie-charts or number lines (e.g., into halves and/or fourths) may be presented to scaffold how the areas/locations of $1/2$ and $2/4$ are aligned.

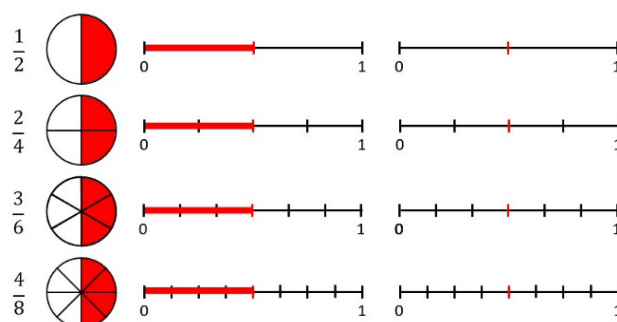


Figure 1: Fraction representations. Symbolic, part-whole, magnitude as length, and as a point on the number line (from left to right)

1.2 Game Elements

For a more comprehensive understanding of the game-based learning literature as well as the added complexity of designing a learning game, it is crucial to consider the fundamental components of a game (cf. Plass et al, 2015). The foundation of each game is its *game mechanics*, which determine player interactions with the game, such as the way a game avatar is controlled/can be moved/etc. In addition, *visual* and *musical* art are used to convey information about the game environment through the senses, while *incentive systems* like points or collectibles are implemented to motivate players. Additionally, the game's *narrative* ties the gameplay together into a cohesive story. These five elements (game mechanics, visual and audio aesthetics, incentives, narrative) can be combined in various ways to create a game, depending on the resources and design available. However, for games specifically focused on educational purposes—and in our case, games for fostering fraction understanding—the educational content is an additional crucial element. This content must be effectively conveyed through all the other game elements to support learning. While some research has criticized the use of game elements in education for potentially distracting students if not done properly (Mayer, 2020), other studies suggest that aligning game and learning mechanics can lead to successful learning outcomes through *intrinsic integration* (Habgood and Ainsworth, 2011). But what about existing games fostering fraction learning?

1.3 Game Elements Fostering Fraction Understanding

The relevance of intrinsic integration was particularly highlighted by a systematic literature review on games for fostering rational number understanding (Kiili et al, 2019). The review indicated that there is an apparent lack of intrinsic integration in most of the existing games to foster fraction understanding, meaning that the learning content of fractions was not integrated effectively into the gameplay through the game mechanics of the respective games. Nevertheless, number line games were most prevalent in the review, with the mechanics

including ordering, comparing, and estimating—depending on the game and the tasks covered in the respective studies. Players typically have to estimate the location of a fraction on the number line (see Figure 2 for a generic illustration of elements used in such games), or otherwise work with fractions and their magnitudes, which often appear in their symbolic a/b (often along their decimal or percentage forms, i.e., 0.5, 50%) but also non-symbolic representation (e.g., pie-charts, area models, etc.).

Operating upon the number line—the characteristic feature of such games—is one more element they share. In particular, the movement mechanics, either of an avatar or other physical objects that are manipulated by the player. Occasionally, this movement is accompanied with a jumping or bouncing ability which expands to other systems (e.g., jumping over traps or as a way to estimate a fraction’s magnitude upon landing). As highlighted in the review, *feedback* is another very important occurring element in these games. *Immediate feedback*, when players move their in-game avatars, by updating their location, as well as feedback upon whether their estimation was close or far from the position of the target fraction on the number line: Depending on the game and the difficulty setting (Gresalfi et al, 2018; Ninaus et al, 2017), the game will indicate various degrees of success. For instance, more incentives offered when closer to the estimation or hints that will be given for extra attempts for estimating the same fraction—to scaffold the learning process as well as to adapt the progression of the game to players’ needs and performance (e.g., game avatar being happy/sad if close to their estimation).

Additionally, *corrective feedback* is used in a lot of these fraction number line estimation games indicating the correct position of the respective fraction on the number line after the player provided their estimation. Common gameplay loops involved in these games are: i) a target fraction is presented on the screen to be estimated by the player (e.g., “ b ” in Figure 2). After the player estimates its position on the number line, ii) another target fraction appears, is estimated, and so on and so forth. In case the game is designed with corrective feedback, after the player estimated the respective target fraction, its correct position on the number line is indicated by the game (e.g., by a short vertical mark, revealing the hidden position, e.g., “ f ” in Figure 2). This seemingly simple game design reflects the mathematical learning content intrinsically, as it represents fraction magnitude as a relative length on the number line (i.e., “ $3/4$ goes here on the number line”), as well as the distance between players’ estimation and the correct position (e.g., “you were this far off/close”).

The effect of feedback may be magnified when accompanied by other mechanics—such as virtual incentives. For instance, a point-incentive system is often used awarding points based on estimation accuracy and thus directly linked with the above-mentioned degrees of success—to increase players motivation. Moreover, a more summative overall grading is often used, such as awarding stars for reaching a certain threshold (e.g., upon level completion with a certain mean accuracy/a certain number of points awarded etc.)—which seems to be especially appealing and driving for young players (Thoma et al, 2023). Finally, the game elements are typically embedded in a narrative to tie them together in a coherent story to immerse the game play. For example, a dog digging for bones that a cat has hidden along a number line (e.g., Koskinen et al, 2023), a hero digging for gold coins (e.g., Kiili et al, 2018), or a ball bouncing higher with every correct estimation (Riconscente, 2013).

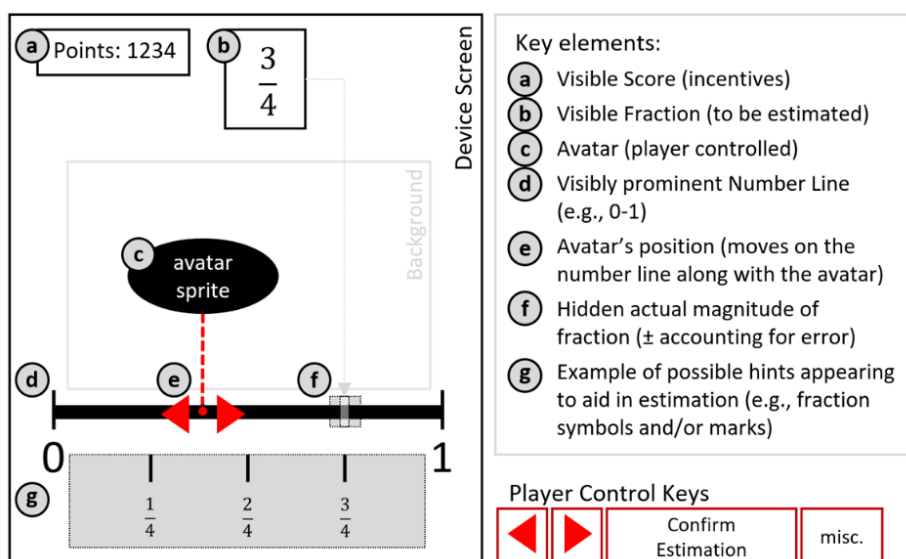


Figure 2: Generic illustration of a fraction number line estimation game

In sum, through the i) integration of game mechanics (i.e., movement on the number line, various levels of feedback), ii) clear visual representations (i.e., position of avatar reflecting fraction magnitude on the number line, corrective feedback), iii) a narrative (e.g., gold or bones hidden in positions on the number line), iv) incentives (embedded as measure of estimation accuracy, extrinsic star system) all these link to—and are designed based on—the mathematical content of fractions (reflected as magnitudes on a number line).

2. The Current Prototype

Considering the aforementioned, the aim was to further develop and combine above-described game elements to create a new number line estimation game that—among others—facilitates a comprehensive understanding of fractions to better illustrate fraction equivalence. In this manuscript, the focus is on the central game mechanic we developed to foster the integration of the fraction concepts of part-whole relations and magnitude to then facilitate understanding of fraction equivalence. As mentioned earlier, available games or apps (e.g., Gresalfi et al, 2018) typically approach the aspect of fraction equivalence primarily via the identification and association of fractions (e.g., $1/2$) with their equivalent counterparts in a list (e.g., $1/3$, $2/4$, $2/6$, etc.) or other representational formats, such as pie-charts, by selecting, matching or pairing them together. From a mathematical perspective these are indeed the fundamental skills related to fraction equivalence. On the other hand, these actions are usually not well-integrated with other fraction concepts in games. This might prevent a comprehensive understanding of fractions across concepts like part-whole, magnitude, equivalence, etc. and in turn facilitate common misconceptions mentioned in the literature.

Firstly, we conceptually introduce the new game mechanic allowing us to better integrate the concepts of part-whole relations and magnitude of fractions—called for descriptive simplicity, the *wheel of fractions*. We later outline how it can be applied to specifically facilitate the concept of fraction equivalence. For both aspects, we will first describe the conceptual background before outlining its prototypical implementation in a game. The prototype's latest version is available at: <https://spoonsweet.itch.io/felis-fractus>.

2.1 Wheel of Fractions—Conceptual Background

2.1.1 Length

To better understand the mechanic used in the game it is necessary to introduce the underlying conceptual idea. The important aspect that allows the wheel of fractions to work as a transition tool between part-whole relations and magnitude, is the dimension of length. Length, appears in two forms: i) as the circumference of the wheel, and ii) as distance from 0 on the number line (Figure 3). Hence, a design is required where the wheel's circumference matches the length of the number line which it rolls along. Consequently, as the wheel rolls along the number line, any part of the circumference is equivalent to the respective length on the number line (e.g., rolling the wheel for $3/4$ of the circumference will take you $3/4$ of the number line, Figure 3a).

2.1.2 Area

Transferring this idea to area models is important for visual communication. As mentioned in the literature, it is usually sectors of the wheel that represent fractions as a pie-chart, as these are visually more distinguishable than their arcs. Therefore, in the wheel mechanic, it is important to showcase this representation of sectors as parts of the whole of the wheel. The dynamic use of colour helps illustrate the transition animation, (as depicted in Figure 3b,c,d).

2.1.3 Segments-spikes

Another bridge between the two representations becomes evident by introduction of the circle radii. Radii are typically added to showcase a fraction as segments, meaning parts of a whole, for instance, “3 shaded parts out of 4” (Figure 3c). Comparable to the segmentation of the wheel through radii, it is possible to segment the number line. For the prototype game mechanic, the length of the radii is extended to create a spiked wheel. By rolling the wheel, these protruding spikes puncture the number line, segmenting it into equal parts (Figure 3d).

To conclude, the rolling animation of the spiked wheel, allows the visual and mathematical integration of the fraction concepts of part-whole relations and magnitude on the number line, at the same time carrying over all the supporting elements of length, area, radii/segments.

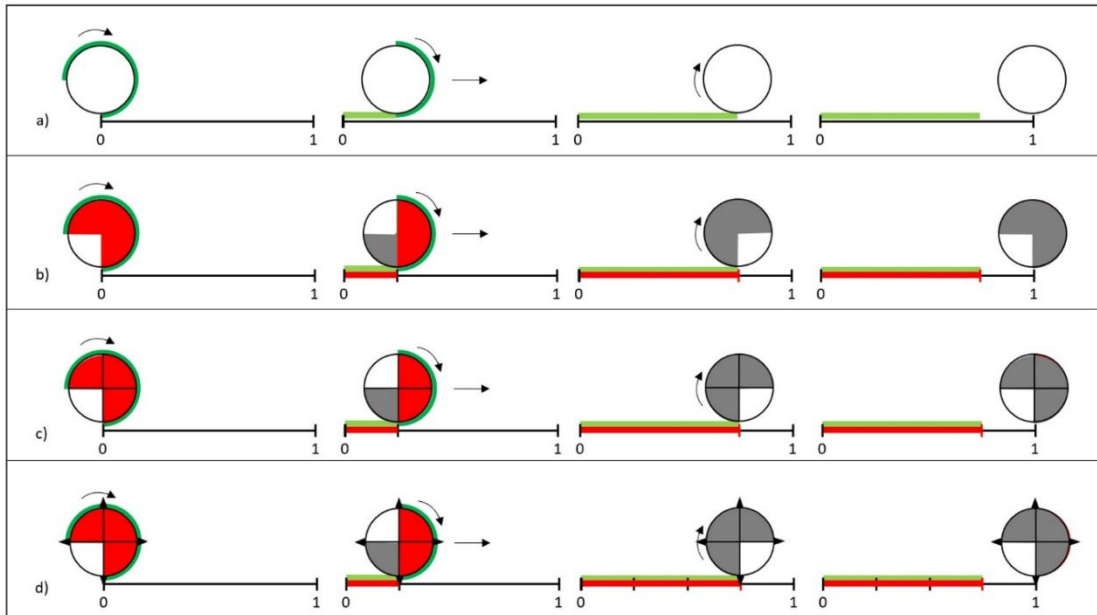


Figure 3: Schematic illustration of the wheel rolling. a) Example 3/4, of the conceptual idea with the circumference turning to length, b) transition of area/colour as the wheel rolls, c) radii and sectors, d) the spikes segment the number line by rolling across

2.2 Wheel of Fractions—in a Game

Designing and prototyping the wheel system in a game, allows for a variety of in-game tasks and affordances, some of which are presented here. All the above-mentioned parts of the system are used in the game, either separate or in an interconnected way. In the following, we first describe the implementation of the wheel of fractions mechanic conceptually, expand on more complex systems (Figure 4), before we present first playable prototypes (Figures 5 and 6).

2.2.1 Dynamic segments and systems of equivalence

The initial creation of the wheel system was to accompany the original number line fraction estimation tasks (e.g., Figure 2), by offering a dynamic segmentation system to the player. By manipulating the number of spikes in their wheel, players can partition their number line in equidistant segments, which aid them in future estimations. As already seen in Figure 3d, a wheel with four radii, would segment the number line in quarters. If a player wishes to change the segmenting to fifths, they can retract the wheel, add an extra spike, and roll it again. For visual clarity, upon retraction of the wheel, the old marks are cleared from the screen.

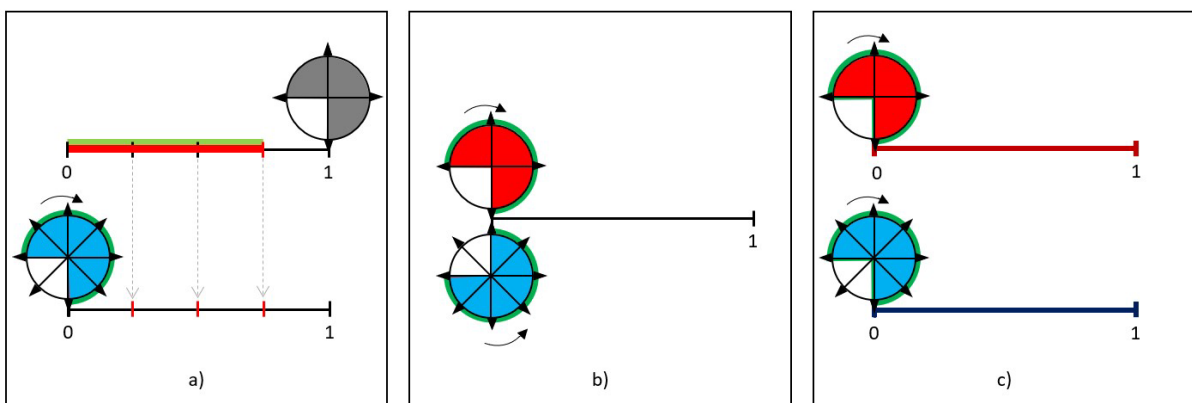


Figure 4: Equivalence with wheel systems. a) One after the other two wheels are rolled on the same number line, b) one number line is punctured simultaneously from top and bottom, c) two wheels are rolled on two parallel number lines

However, there also are ideas being explored where past segments leave a trace (e.g., with a lower opacity, different colour/shape) allowing the comparison of segmentations and magnitudes. For example, a series of red punctures could signify the segmentation in quarters while blue punctures could reflect eighths (Figure 4a). As

one of the main aims is the focus on fraction equivalence, this would assist in illustrating that equivalent fractions share the same magnitude. The panels in Figure 4 depict this idea in three different implementations. Using the magnitudes of the above Figure 3 example, different colours are used here to help with its visualization. In each panel, a different system is explored. In Figure 4a, the wheel has been rolled once, and the number line segmented in quarters. Leaving the punctured segments in-place, the wheel is retracted and armed with a total of eight spikes. When rolled for the second time, some of the new spikes—specifically the ones at $2/8$, $4/8$, $6/8$ —would fit at the already existing punctures, from the quarter segmentation. Figure 4b, shows a combined system of one number line being punctured from top and bottom simultaneously, essentially creating paths at the equivalent positions—since punctured at both ends. Finally, in Figure 4c, two parallel number lines are positioned in such a way, that equivalent fractions would be aligned in their respective positions.

2.2.2 Punctures and their usage implemented in a game

Implemented in a game, each puncture on the number line offers a visible mark (Figure 5a,b). This can be used by players in the ways mentioned above (as segmentation of the whole and connecting equivalent fractions), but also allows for existing game mechanics to be enriched. For example, as noted in the literature, fraction magnitude is usually estimated by the player, by moving an avatar to a specific location on the number line starting from 0 (e.g., Figure 2, or Figure 6 for the prototype estimation). This can be enhanced with a new mechanic, where the avatar is designed to drag a ribbon or elastic band from the 0-mark with the option of securing it in any of the puncture-marks. Thus, the respective magnitude may be reflected explicitly (Figure 5b). With this action, the visualization gives the magnitude on the number line a second dimension, i.e., length, in addition to being a point on the number line (as shown in Figures 1 and 2). This dragging mechanic gives the opportunity for players to “shade-in” the number line, and experience magnitude as length from 0 to the desired position. Depending on the actual task design, this affordance may be implemented as just described (Figure 5b) or be integrated with the wheel system, unravelling automatically (as indicated in the abstract Figures 3 and 4a).

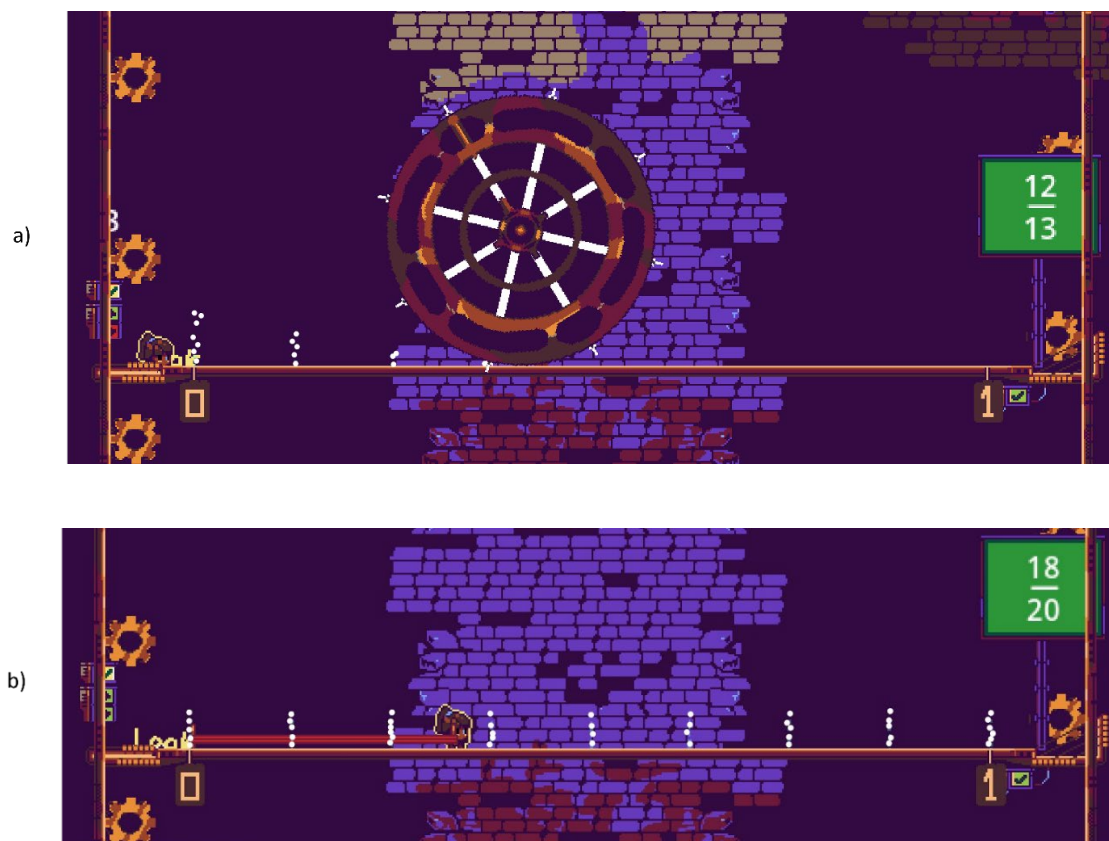


Figure 5: *Wheel of fractions* in-game. a) Armed with 8 spikes, the wheel rolls across the number line. b) Dragging the elastic band mechanic. From 0 towards the $3/8$ puncture

2.3 Supportive Game Elements

2.3.1 Spikes as a resource

Acquiring spikes to add on the wheel can be directly linked to the incentive and narrative game elements. In our case—building upon the foundation of a fraction number line game—the concept of point-incentives for successful estimations can be embedded mechanically and narratively. For instance, estimating a fraction magnitude successfully (e.g., with $\leq 2\%$ estimation error) would reward the player, not only with score points, but also with a high-value in-game artifact. This collected artifact, could then be used to form a spike to arm the wheel. For example, playing the game, a player estimates fractions with varied levels of success. At some point, they manage a successful estimation above the threshold (e.g., $\leq 2\%$) and are awarded with an artifact. After it is formed to a spike, it arms the wheel, the wheel is rolled, and the player now sees how the wheel system works. After a second successful estimation, the wheel is armed with another spike, rolled over, and the number line is segmented into halves. Now the player can estimate fractions more accurately since they have a clear reference of 0.5. With more successful estimations, more spikes are collected and can be used on the wheel, to segment the number line, allowing for more targeted estimations, which should lead to more successful ones—and eventually—more spikes. Interconnecting the two systems this way creates two gameplay loops that feed into each other: i) estimating fractions to collect spikes, and ii) arming and rolling the wheel to improve future estimations. This interconnectedness should keep gameplay interesting, add complexity, give agency to players (as their actions have impact and direct improvement to their future estimations), and allow for strategy (of choosing which and how many reference points), all of which are beneficial aspects known to keep players motivated and continue playing (Plass et al, 2015).

2.3.2 Overarching narrative

The above-described game mechanics and elements only become fully effective when integrated into an overarching narrative that incorporates these different aspects into one coherent game. The idea for this narrative is as follows. The game takes place in an underground sewer system. At the bottom of the sewer system, in the acid waters, laser-cats swim (Figure 7). The player, through their avatar, will be invited to restore and redevelop the underground village civilization. To do so, players will have to collect energy from the laser-cats. When excited, a laser-cat shoots up from the depths in a straight line (Figure 6b and 7a). If the player has proper equipment (i.e., specific in-game thresholds have been reached), they can capture energy from the cats by estimating the position they are going to shoot up on the number line. Captured cats can later be repurposed as an energy source in a lab, reinforcing the narrative and creating more gameplay loops.



Figure 6: The prototype. a) Basic elements of a fraction estimation number line game. b) Corrective feedback indicates the position of the $\frac{3}{8}$ fraction magnitude

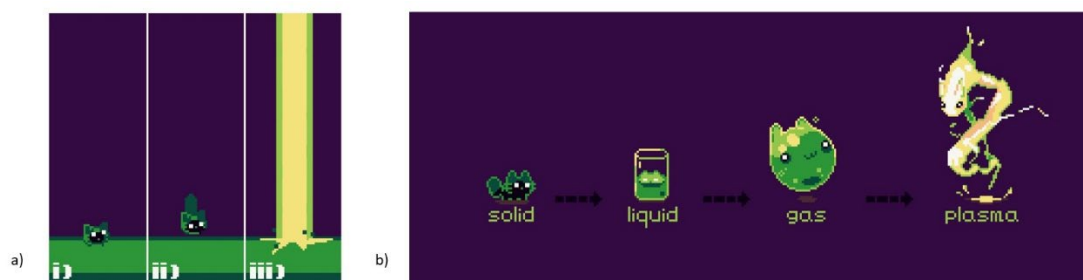


Figure 7: Laser-cats. a) Animated cat's laser trajectory as corrective feedback/magnitude on the number line, b) states of cat matter concept art

Aiming for intrinsic integration, each activity taking place in the game is designed towards fraction understanding and especially aiming towards fraction equivalence. As such, laser-cats' existence is multifaceted and embedded to all the aforementioned systems and mechanics. Firstly, as in-game objects, each cat spawns with a fraction magnitude between 0-1. This magnitude will be descriptive of each cat, and accompany it in the game through the various tasks. On the number line, from a mathematical perspective, a laser-cat's trajectory is vertical to the platforms (Figure 6b) working as corrective feedback on fraction number line estimations. Their laser properties allow them to fire upwards throughout the sewer system, with the possibility of simultaneously hitting numerous parallel platforms, assisting in fraction equivalence (e.g., as described above in Figure 4c). Cats are also incentives on the number line. If the player estimates their position correctly—meaning their avatar is literally inside the blast zone (Figure 6b)—energy points are gained. In case the estimation is successful (e.g., $\leq 2\%$), the cat is also captured as an artifact—and can be armed as a spike for the wheel (Figure 5). Otherwise, if taken back to the village, can be used for other narrative purposes after being stored as batteries in bottles (Figure 8c). This bottling of cats, works through a mini-game, which awards and reinforces fraction representations: in lab equipment screens, the cat's magnitude will appear symbolically, then on a short number line, and after that, on a pie-chart (Figure 8a,b), whereupon the player will have to confirm the representation with simple button presses. This way the cat will be stored as a battery, and simultaneously registered in a bestiary. This will reinforce not only integration of the different representation formats, but also allow for an in-game encyclopaedia showcasing all the fractions captured. As such, equivalent fractions will be logged in the same entry (e.g., $5/11 = 10/22$), considering additional game elements to increase mathematical depth.

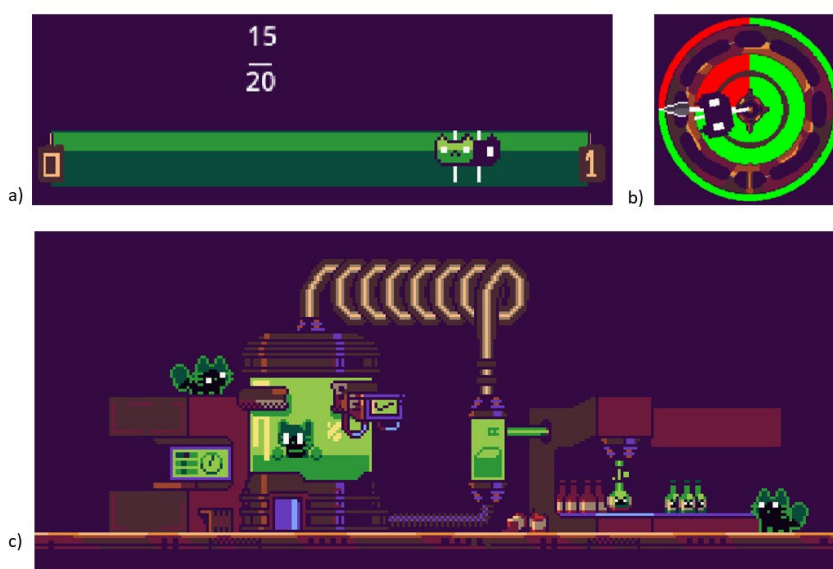


Figure 8: Bottling cats mini-game. a) The player has to align a moving mark with the desired fraction magnitude on the number line, b) and a rotating mark on the pie-chart, c) concept art of the lab.

3. Discussion

The aim of this article was to introduce a new game mechanic allowing for the integration of fraction concepts of part-whole and magnitude as well as applying it to facilitating the understanding of fraction equivalence. This

wheel of fractions—synthesized as a wheel whose circumference matches the length of an associated number line—reflects part-whole relations as known from pie-chart representations. While common in introducing fractions, pie-charts were also found leading to some misconceptions about fraction magnitude. Thus, we aimed at facilitating the transition between this pie-chart representation and the concept of magnitude as represented on a number line—a representation format known to be fostering number magnitude understanding. By extending the radii that segment the wheel, spikes are formed. When the spiked wheel is rolled across the number line it punctures and thus segments the number line into equal parts allowing for an integration of the fraction concepts of part-whole and magnitude. Embedding the wheel mechanic in a fraction number line estimation game further allows for illustrating equivalent fractions—for which, different numbers of spikes puncture the number line at the same position. This gameplay loop may be fuelled by other game mechanics such as virtual incentives and an overarching narrative corroborating the mathematical richness of the game. Even though, the conceptual idea of the wheel is of compelling simplicity and apparent validity, empirical studies are needed to evaluate the usability of the game—and its specific game elements—as well as its educational effectiveness in terms of fostering fraction (equivalence) understanding.

References

- Devlin, K. (2011) *Mathematics education for a new era: Video games as a medium for learning*. CRC Press.
- DeWolf, M. and Vosniadou, S. (2015) “The representation of fraction magnitudes and the whole number bias reconsidered”. *Learning and Instruction*, 37, pp.39-49. <https://doi.org/10.1016/j.learninstruc.2014.07.002>
- Fazio, L. and Siegler, R.S. (2011) *Teaching fractions* (Vol. 22). North Coburg: International Academy of Education.
- Fuchs, L.S., Malone, A.S., Schumacher, R.F., Namkung, J. and Wang, A. (2017) “Fraction intervention for students with mathematics difficulties: Lessons learned from five randomized controlled trials”. *Journal of Learning Disabilities*, 50(6), pp.631-639. <https://doi.org/10.1177/0022219416677249>
- Gresalfi, M.S., Rittle-Johnson, B., Loehr, A. and Nichols, I. (2018) “Design matters: explorations of content and design in fraction games”. *Educational Technology Research and Development*, 66, pp.579-596. <https://doi.org/10.1007/s11423-017-9557-7>
- Habgood, M.J. and Ainsworth, S.E. (2011) “Motivating children to learn effectively: Exploring the value of intrinsic integration in educational games”. *The Journal of the Learning Sciences*, 20(2), pp.169-206. <https://doi.org/10.1080/10508406.2010.508029>
- Ke, F. (2008) “A case study of computer gaming for math: Engaged learning from gameplay?”. *Computers & education*, 51(4), pp.1609-1620. <https://doi.org/10.1016/j.compedu.2008.03.003>
- Kiili, K., Koskinen, A. and Ninaus, M. (2019) “Intrinsic integration in rational number games—a systematic literature review”. In *International GamiFIN conference* (pp. 35-46). CEUR-WS. <https://ceur-ws.org/Vol-2359/>
- Kiili, K., Moeller, K. and Ninaus, M. (2018) “Evaluating the effectiveness of a game-based rational number training-In-game metrics as learning indicators”. *Computers & Education*, 120, pp.13-28. <https://doi.org/10.1016/j.compedu.2018.01.012>
- Koskinen, A., McMullen, J., Ninaus, M. and Kiili, K. (2023) “Does the emotional design of scaffolds enhance learning and motivational outcomes in game-based learning?”. *Journal of Computer Assisted Learning*, 39(1), pp.77-93. <https://doi.org/10.1111/jcal.12728>
- Mayer, R.E. (2020) “Cognitive foundations of game-based learning”. In Plass J.L., Mayer R.E., and Homer B.D., (eds), in *Handbook of game-based learning*. The MIT Press. pp.83–110.
- Ninaus, M., Kiili, K., McMullen, J. and Moeller, K. (2017) “Assessing fraction knowledge by a digital game”. *Computers in Human Behavior*, 70, pp.197-206. <https://doi.org/10.1016/j.chb.2017.01.004>
- Pan, Y., Ke, F. and Xu, X. (2022). A systematic review of the role of learning games in fostering mathematics education in K-12 settings. *Educational Research Review*, p.100448. <https://doi.org/10.1016/j.edurev.2022.100448>
- Plass, J.L., Homer, B.D. and Kinzer, C.K. (2015) “Foundations of game-based learning”. *Educational psychologist*, 50(4), pp.258-283. <https://doi.org/10.1080/00461520.2015.1122533>
- Riconscente, M.M. (2013) “Results from a controlled study of the iPad fractions game Motion Math”. *Games and Culture*, 8(4), pp.186-214. <https://doi.org/10.1177/1555412013496894>
- Sidney, P.G., Thompson, C.A. and Rivera, F.D. (2019) “Number lines, but not area models, support children’s accuracy and conceptual models of fraction division”. *Contemporary Educational Psychology*, 58, pp.288-298. <https://doi.org/10.1016/j.cedpsych.2019.0>
- Siegler, R.S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M.I. and Chen, M. (2012) “Early predictors of high school mathematics achievement”. *Psychological science*, 23(7), pp.691-697. <https://doi.org/10.1177/0956797612461752>
- Thoma, G., Bahnmueller, J., and Moeller, K. (2023) “Problem solving when digging for fractions: a case study of a fraction number line game”. In I.Papadopoulos and Patsiala N., (eds.), in *Problem Solving in Mathematics Education: Proceedings of the 22nd ProMath Conference*. Thessaloniki. pp.81-95. http://promath.org/docs/ProMath22_Proceedings.pdf
- Van Eck, R.N., Shute, V.J. and Rieber, L.P. (2017) “Leveling up: Game design research and practice for instructional designers”. *Trends and issues in instructional design and technology*, 227, p.285.