The Game of Dominoes as Teaching-learning Method of Basic Concepts of Differential Calculus

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Abstract: This paper will present teaching-learning mathematics strategies, specifically differential calculus in a variable, through a fun and competitive space in which action is manifested by the student and not only the repetition of information by the teacher. This action refers to motivate, problematise, summarise, and coordinate a dominoes game whose thematic cards are designed around the basic and main contents of differential calculus. The strategies for teaching this area are diverse and precisely the game of dominoes is one of the most used strategies in the practice of mathematics because it stimulates logical reasoning and mental skills. The objective is to identify the way in which the game of dominoes affects the learning and understanding of fundamental concepts of differential calculus in a variable by means of experimentation carried out on students of the first semester of the School of Engineering and Sciences of Technological of Monterrey. Finally, the results of this study will be presented and the use of this strategy in other topics around mathematics will be recommended to facilitate logical and meaningful learning in students.

Keywords: Collaborative Learning, Educational Innovation, Higher Education, Game-based Learning, Multiple Intelligence, Logical-Mathematical Intelligence.

1. Introduction

The “game-based learning” is an educational approach that uses games as the primary tool for teaching and learning. This method is based on the premise that students learn more effectively when they are actively engaged and having fun and also has been applied in various areas and has generated a variety of fun and committed experiences. A game designed under this methodology is known as “serious game”, which are games whose design has a more educational purpose than entertainment. Moreover, this type of game makes the user explore what interests him and allows him to formulate hypotheses, to suggest and even to create new situations without any imposition. Rajkovic (2020) and Laski (2014) states that the game encourages and develops an attitude with respect to the use of intelligence. Through this, it is possible to develop intelligence in an easier way and many other skills and abilities can be enhanced. In addition, through the game, teaching ceases to be tedious and becomes a strategy of use and discovery of knowledge where multiple skills and abilities are developed and strengthen the integral formation of students. Muñiz (2014) states that learning mathematics can be a motivating experience if we base it on constructive and playful activities. The use of games in maths education is a strategy that allows students to acquire skills in a fun and attractive way.

Specifically, in Engineering, one of its main branches is Mathematics, in which great mental abilities are developed that allow solving problems and making appropriate decisions that occur daily. Within this branch, is the Differential Calculus which, in general, presents a series of difficulties in terms of its learning and conceptual understanding, since students show great difficulty in developing it and, adding the routine strategies used by the teacher, allow the appearance of barriers, which inhibit students from accrediting courses related to this area.

Play a dominoes whose tiles represent conceptual contents of the Differential Calculus, would allow students to immerse a scenario where, in addition to articulating corresponding tiles, they can listen to recommendations from the teacher and even from their peers and thus generate a debate among them in a collaborative, critical and constructive way, which implies the generation of effective environments of collaboration and negotiation in multicultural contexts with respect and appreciation for the diversity of knowledge and, in turn, allows to integrate different types of reasoning in the analysis and solution of problems, with readiness to continuous learning.

The purpose of this research is to propose a different strategy to understand basic concepts of Differential Calculus through the didactic game of dominoes. This new strategy is intended to complement the exclusive and permanent attention of the student with what the teacher explains since, in general, the transfer of knowledge between teacher and students is carried out through routine strategies.

To answer the research question, an experiment was carried out at the Technological of Monterrey with some students from the first semester of the School of Engineering and Sciences.

2. Methodology

2.1 The Focus of the Research

The present research focused on measuring the ability to understand basic concepts of differential calculus since understanding this area of mathematics is considered indispensable for any engineering student. Generally, students entering the first semester of any engineering career need to improve in understanding concepts related to the derivative of a function of a variable.

For this reason, this article is based on a pure experimental design which consists of manipulating one or several independent variables to observe changes in dependent variables in a controlled situation; that is, it is intended to establish the possible effect of a cause that is manipulated. Moreover, the research design incorporates the application of a pre-test and a post-test to experimental and control groups. Specifically, the pre-test was applied on the first day of class in order to measure and diagnose the level of conceptual understanding of students about some important issues of the derivative, such as, constant change rate, critical points, the monotony of functions, concavity of the graph of a function, among others, and also a post-test was applied at the end of the subject in order to measure the level of understanding of the concepts above again. It is important to note that, between the application of the pre-test and the application of the post-test, five weeks pass, in which the teacher explains all the topics related to differential calculus and applies the game of dominoes several times.

2.2 Research Type

The present research was developed under the focus of action research, which allows using various strategies in the professional practice of teachers to provide an improvement. In this sense, Elliot (1993) establishes that "Action research is a methodological option of great richness since, on the one hand, it allows the expansion of knowledge, and on the other, it gives concrete answers to the problems that the participants of the research are raising, which, in turn, become co-researchers who actively participate in the entire research process as a result of the constant reflections that are promoted in this process".

According to this, the use of the domino game is proposed as a didactic strategy to improve the understanding of basic concepts of Differential Calculus in a variable and reduce the difficulties presented by students in this process.

2.3 Population

The population that participated in this study was 90 students, all from the first semester of the School of Engineering and Sciences of the Technological of Monterrey, with aged between 18 and 19 years. Of the total number of students, two groups (30 students each) formed the experimental groups, and 30 students formed the passive control group. Only in the two experimental groups was applied the dominoes game. The designation of the groups was carried out by the teacher randomly.

2.4 The Stages of the Research

2.4.1 Stage 1. Literature Research

A documentary search was conducted, nationally and internationally, on previous research on the application of didactic games as a strategy for learning mathematics to generate a characterization of the problems presented in those previous investigations.

2.4.2 Stage 2. Diagnostic (pre-test)

At this stage, a multiple-choice diagnostic exam was designed consisting of 5 questions related to fundamental concepts of differential calculus, such as, constant change rate, increasing and/or decreasing functions, concavities of a graph, among others, and whose duration was 20 minutes. The pre-test allows instructors to assess the understanding and to measure the knowledge existing of Differential Calculus concepts in the students. The exam was applied to the three research groups (two experimental groups and one control group). The following table describes the contents of the diagnostic test reagents and their respective educational intentions.

Table 1: Description of diagnostic test questions

Reagent	Content	Didactic intention
1	Graphical modelling of the constant average rate of change.	Identify errors in the graphing approach related to the meaning of the constant rate of change.
2	Description of the constant rate of change.	
3	Monotony of functions	Manipulation of mathematical language to express the definition of increasing functions.
4	Critical points of a function graph.	Conceptualization of a critical point from the verbal point of view.
5	Inflection points and concavity of the graph of a function.	Conceptualization of a point of inflection and understanding of concavities of the graph of a function.

2.4.3 Stage 3. Design of the dominoes

De Guzmán (1984) advises using games that present advantages in the psychological and motivational aspects compared to mathematical contents. For this author, the advantages of this type of game do not require lengthy or complicated introductions since what is sought is fun and the possibility of getting into action quickly. Many mathematical concepts, even some intense ones, can have a simple introduction and a possibility of action with very ingenious instrument designs. Cuevas (2013) defines cognitive tools as the acquisition of mathematical knowledge mediated by the action of a material or symbolic tool. In the same sense, dominoes turn out to be a handy tool of this kind for teachers who teach mathematics. The dominoes described below consist of 28 tiles, each with two parts. The difference from the traditional dominoes game we play is that, in this, the objective when joining one tile with another is valid as long as the tiles match, while in our case study (mastered maths), the cards must represent the same mathematical concept. For this reason, for the design of the game cards, the research team selected seven fundamental concepts of differential calculus that students in the first semester of Engineering should understand.

- Constant average rate of change (tokens representing 0).
- Monotony of functions (tokens representing 1).
- Critical points (tokens representing 2).
- Optimization of functions (tokens representing 3).
- Inflection points (tokens representing 4).
- Concavities of the graph of a function (tokens representing 5).
- Derivation rules (cards representing 6).

Once the fundamental concepts have been defined, seven propositions were defined for each one, equivalent to each other and the respective fundamental concept. Subsequently, the game cards were formed by combining each concept's propositions (without repeating them). It is essential to clarify that to define propositions equivalent to the fundamental concept of critical points, only soft functions were considered to avoid defining propositions related to the domain's points where the function's derivative does not exist. Below is an example of how the 28 tiles of the game were formed.

Table 2

Constant average rate of change	Constant average rate of change
The function graph is a line	Monotony of functions
$x \in Dom(f): f'(x) = 0$	$\frac{\Delta y}{\Delta x} = c, where\ c\ is\ a\ constant$
The change of the dependent variable is a multiple of the change of the independent variable	Point on the graph where the function goes from decreasing to increasing or vice versa

They are critical point for the f'' graphic.	The change in the dependent variable and the change in the independent variable are directly proportional
The instantaneous rate of change approaches a constant	$f''(x) > 0 \ o \ f''(x) < 0, \forall x \in I$

The importance of first analysing the game and placing the dominoes correctly instead of just doing it mentally stands out in some studies on brain function. Hannaford (2008) emphasizes that using hands to activate neural circuits to facilitate learning is extremely necessary. Moreover, Hannaford (2011) recommends a sensory environment with many natural stimuli for the ear, touch, smell, and sight. In this sense, Gardner (2014) indicates that it is necessary to extend the concept of intelligence to the concept of multiple intelligences. In this way, the use of a board game as a dynamic didactic tool undoes the traditional way of learning, that is, to be only listening and seeing what the teacher explains.

2.4.4 Stage 4. Game Implementation

During the implementation of the game, each experimental group was subdivided into teams of 4 students at the same table. The teacher explained the rules of the game consisting of the following:

- The game consists of 28 tiles, and all are placed face down on the table.
- Each of the 4 students at the same table randomly select 7 titles of the game.
- The player who starts the game is drawn between the same students who make up the same game table.
- When placing a tile, all students involved at the table must agree that the tile placed is correct and matches the corresponding concept. At this moment, the critical and constructive discussion between the students at the same game table is open.
- If the tile placed by a student on their first attempt is correct, then the player who placed it gets a correct answer and a score of +4.
- If the tile placed by a student on their first attempt is not correct, then the group discussion is opened, a score of -2, but that player can place another tile correctly without counting any positive scores on the second attempt.
- If the student does not have any tile that match with the concepts of the other tiles on the table, it is counted as a pass, and in this case, a score of 0 should be placed. It is not counted as a correct or incorrect answer, and simply follow the next player's turn.
- All players must record the number of correct or incorrect answers and the respective score on each move.
- The winner will be the first player to run out of tiles or, in case the game is locked, and the winner will be the student with the highest positive total score.

It is essential to clarify that the game was applied as a conceptual review after the professor explained all the issues related to differential calculus in one variable. This stage of the experiment was performed twice in each experimental group; that is, the students of the experimental groups played dominoes twice before the last day of class; where a small competition was planned in these groups, where each table consisted of 4 players (students) and extra scores were awarded in the final grade of the subject according to the score obtained at the end of the game.

2.4.5 Stage 5. Descriptive analysis of the results

Three main moments were identified regarding the implementation of the teaching strategy: before, during and after.

The first moment (the before), which took place on the first day of classes, a multiple-choice diagnostic exam was implemented for the three groups involved in the research, which was previously designed by the teacher and was made up of 5 reagents of 20 points each. The information was collected considering correct and incorrect answers. For each correct answer the student obtained 20 points and for each incorrect answer the student obtained 0 points. The results can be seen in table 2 (for the experimental groups) and in table 3 (for the control group). It is important to highlight that the table shows a continuous measure (exam score) and an ordinal measure that indicates the percentage of students in the group who obtained said score.

Later, during the experiment, the domino game was applied only to the two experimental groups, as a way of

reviewing the contents explained by the teacher; while in the control group, the conceptual review of the content was carried out in a traditional way with some exercises and questions proposed by the teacher. This stage of the experiment was carried out twice after 6 weeks of the academic period had elapsed, with a frequency of 2 weeks; That is, the game was applied in week 7 and week 9 of the corresponding academic period. In addition, a competition was planned between the students of the experimental groups that took place during the last day of classes. In this stage, the teacher collected the information through observations made during the sessions to evaluate the level of participation, collaboration and strategies used by the students to understand the concepts of the game pieces.

In the third and final stage of the experiment, after playing the experimental groups, the three groups again answered the same diagnostic test that was applied in the first moment (with the same number and characteristics of the questions). The information was collected in the same way as it was done in the first moment of the experiment. The results can be seen in table 4 (for the experimental group) and in table 5 (for the control group). It is important to highlight that the table shows a continuous measure (exam score) and an ordinal measure that indicates the percentage of students in the group who obtained said score.

To measure the effectiveness of the strategy and the impact it has on improving learning regarding the understanding of the concepts of differential calculus, the results of the post-test were compared between the experimental group (Who played dominoes) and the control group (Who did not play dominoes) to determine if there is a significant difference. For this, a t-test for independent samples was programmed in Python code with a significance level of 5%, resulting in a p-value of 0.003.

Finally, a perception survey of the implemented teaching strategy was administered to the students with the objective of evaluating their perception of the game, their motivation, and how they believe the game has influenced their learning. The results can be seen in table 6.

3. Results

3.1 Pre-test and Post-test Results

Once the diagnostic tests were completed, a descriptive analysis of the scores obtained in each study group was carried out to observe and measure (before and after the game) understanding some basic concepts of Differential Calculus.

Table 2: Report of the score obtained, on a scale from 0 to 100, from the experimental group during the pre-test.

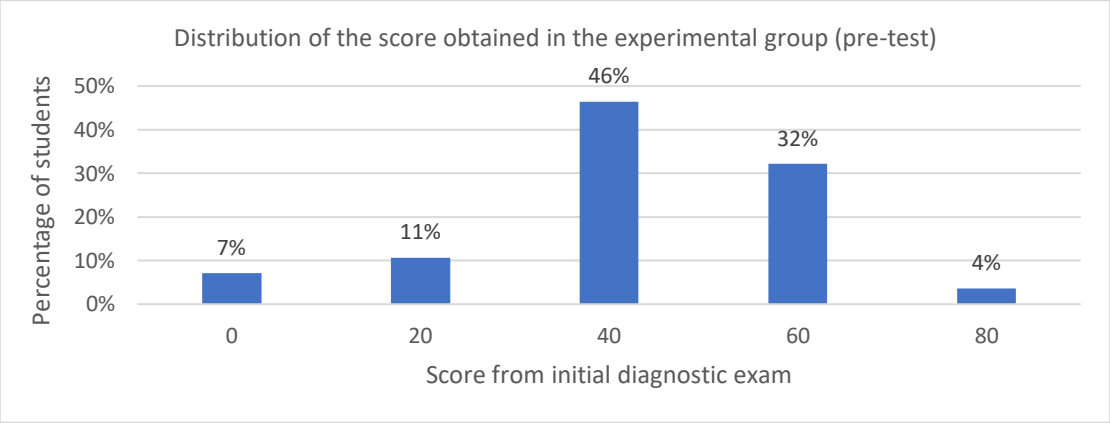

Table 2 shows that only 4% of the students in the experimental group obtained a score of 80 points and can be considered to have a good understanding of the basic concepts of differential calculus. In addition, 64% of students have a very low understanding of these concepts (score less than or equal to 40).

Table 3: Report of the score obtained, on a scale from 0 to 100, from the control group during the pre-test.

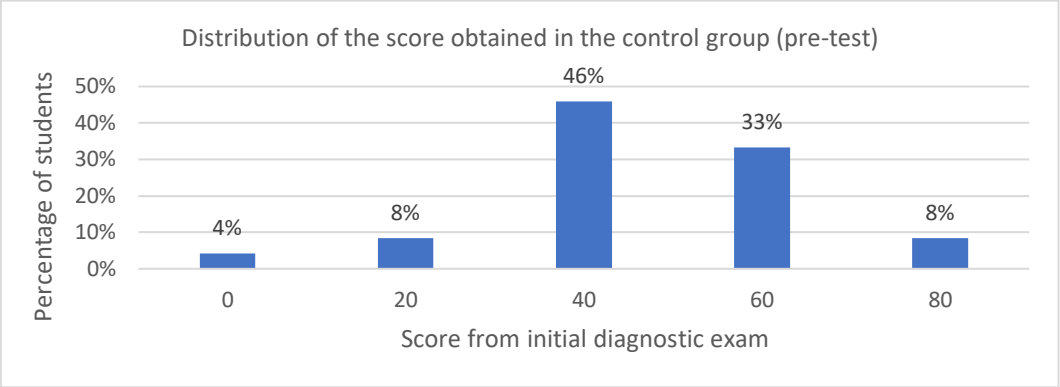

Table 3 shows that only 8% of the students in the control group obtained a score of 80 points and can be considered to have a good understanding of the basic concepts of differential calculus. In addition, 58% of students have a very low understanding of these concepts (score less than or equal to 40).

Table 4: Report of the score obtained, on a scale from 0 to 100, from the experimental group during the post-test.

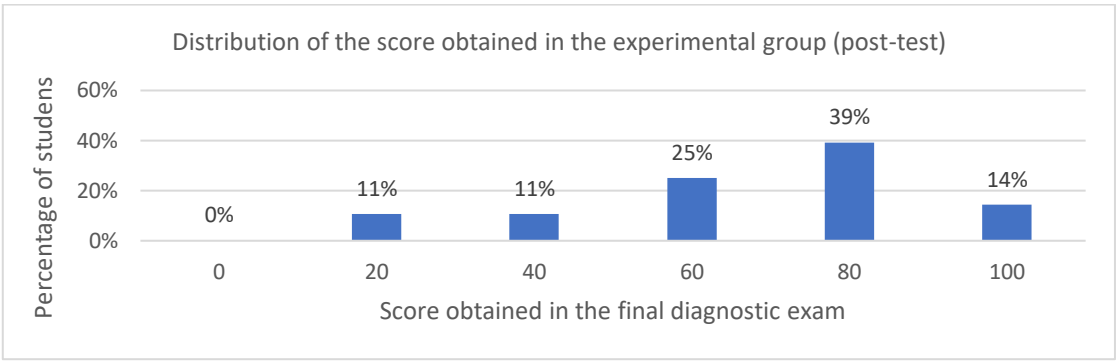

Table 4 shows that 53% of the students in the experimental group obtained a score greater than or equal to 80 points and can be considered to have a good understanding of the basic concepts of differential calculus after having applied the domino game. In addition, only 22% of students still have a very low understanding of these concepts (score less than or equal to 40).

Table 5: Report of the score obtained, on a scale from 0 to 100, from the control group during the post-test.

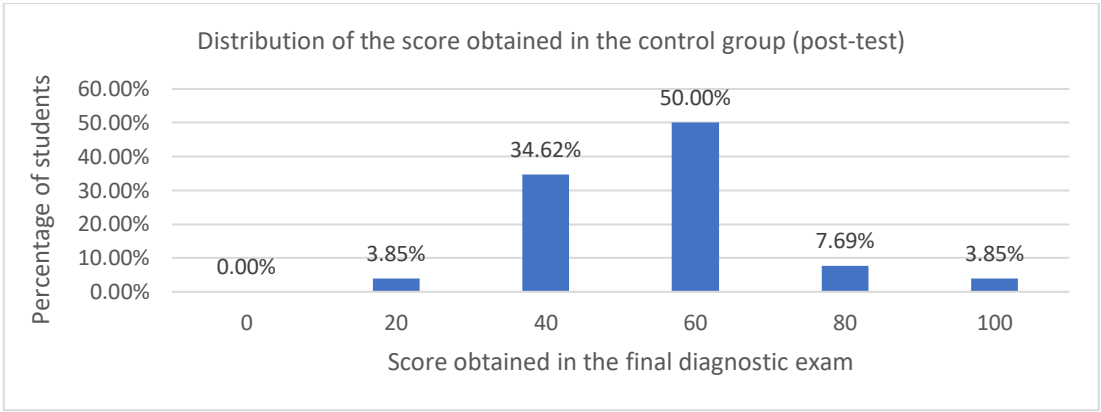

Table 5 shows that only about 11.5% of the students in the control group obtained a grade greater than or equal to 80 points and can be considered to have a good understanding of the basic concepts of differential calculus at the end of the subject and without having applied the game of dominoes. In addition, it can be noted that

approximately 38% of students have a very low understanding of these concepts (score less than or equal to 40) and half of the group has an intermediate knowledge of the concepts addressed.

3.2 Results of the Survey of Student Perception Towards the Teaching Strategy of the Domino

At the end of the experience, a survey was applied to the students of the experimental group on the perception of the didactic strategy of the domino game in order to know, as a group, their impressions about the impact that produces this strategy.

Table 6: Report on the perception and evaluation of collaborative learning through dominoes.

Experimental Group	Criteria and dimensions			
	Participation and respect for opinions	Share responsibility	Mastery of the game	Progress in the understanding of concepts
1	In both groups, all students participate with enthusiasm and without fear which generates an atmosphere of confidence and fun. In addition, there was great respect for the opinions of others and diversity of knowledge.	In several game teams, only 1 student took responsibility to guide the game in a correct way.	In both groups, all students understand the rules of the game and the purpose of the game.	In both groups, the first time the game was applied, several students presented many errors when placing the correct tiles, but at the time of playing for the second time and in the final competition they felt very comfortable, with acceptable handling of concepts and the game flowed faster.
2		In most game teams, several students took responsibility for guiding the game correctly.		

4. Conclusions

Concerning the numerical results obtained, we can note the progress in understanding concepts of differential calculus of the students who formed the experimental group opposite those who formed the control group. Specifically, when comparing tables 1 and 3 of the experimental group against tables 2 and 4 of the control group, we can observe that at the beginning (pre-test), both groups had similar knowledge of such concepts, which can be inferred that knowledge was relatively low. Later, at the end of the explanation of the content by the teacher, after having applied the domino game three times in the experimental group and after applying the post-test in both research groups, a significant improvement is observed in the students of the experimental group concerning the students of the control group in the understanding of concepts of this area of mathematics. It is important to note that the dominoes that were implemented as a teaching strategy in the experimental groups during the two times of practice (before the final competition), had some advantages and disadvantages from the student's attitudinal point of view. As advantages, we can mention that it allows the development of collaborative work and appreciation for the diversity of knowledge. In addition, the teacher's participation was almost unnecessary since, in each team, the support of the more skilled students towards the less skilled was made naturally due to the debate that they themselves generated.

On the other hand, as a disadvantage, it can be mentioned that some students may tend to delegate their participation to others at the game table. It is necessary for the teacher to emphasize that all players should place their tiles without necessarily expecting feedback from their colleagues and to achieve this the teacher will emphasize that before reaching the day of the final competition must study all the concepts handled during class sessions and will not have any supporting material on hand. In addition, having human material like the gaming table teammates and having the dominoes game as a physical activator, allows students to show the knowledge they are acquiring completely live, and it is not necessary to wait for the date of a partial exam that, in addition to other things, generates stress in students. Playing this domino is a way of creating mathematical meaning of what the student imagines when trying to understand the concepts of Differential Calculus by correctly placing the tiles that solve the game. However, as mentioned above, this requires a series of cognitive activities, among which are: training, treatment, and mathematical representations. This didactic strategy involves cognitive activity and attitudinal activity on the part of the students and therefore, it has been

considered collaborative work among the members of the team in order to promote the initiative of the student to place before the card correct. In addition, students' motivation increases considerably in this type of activity, compared to their usual interest during traditional classes. In this way, we have shown an experience whose objective was maintained in the integration of attitudinal, conceptual and procedural components of learning of the Differential Calculus of one variable in students of the first semester of Engineering careers, and we recognize the inclusion of the dominoes game as an appropriate didactic strategy not only for learning the Calculus Differential but for mathematics in general. Finally, through the bibliographical consultation studied before starting the research, it can be contrasted that this type of didactic strategy has a considerable impact on the learning process, and in particular, the dominoes game is a highly valued tool as a technique to help understand concepts and solve difficulties that are presented to the time to study mathematics.

Acknowledgments

The author would like to acknowledge the financial and technical support of Writing Lab, Institute for the Future of Education, Technological of Monterrey, Mexico, in the production of this work.
The author is grateful for the support provided by Dr. Jorge Arturo Garza Venegas, attached to the Science Department, Campus Querétaro, for his valuable collaboration in reviewing the design of the dominoes, specifically the propositions equivalent to the fundamental concepts handled in the game.

References

Chacón, P. (2008). "El juego didáctico como estrategia de enseñanza y aprendizaje. ¿Cómo crearlo en el aula?", *Nueva aula abierta*, Vol. 5, No. 16.

Chamoso, J., Durán, J., García, J. y Rodríguez, M. (2004) "Análisis y experimentación de juegos como instrumento para enseñar matemática", *Suma: Revista sobre la enseñanza y aprendizaje de las matemáticas*, Vol. 47, pp 47-58.

Coronel, J. (2012). *Aplicación de un prototipo de dominó para la enseñanza y aprendizaje de las identidades trigonométricas*. Universidad Popular del Cesar, seccional Aguachica, Colombia. Tesis de pregrado.

Cuevas, C. P. (2013). "El cálculo y su enseñanza", [online], Cinvestav IPN, https://recacym.org/index.php/recacym

De Guzmán, M. (1984*). Juegos matemáticos en la enseñanza*. Universidad Complutense de Madrid.

Elliot, J. (1993). *El cambio educativo desde la investigación-acción*. Ediciones Morata.

Flores, A. (2014) "Enfoque conceptual del cálculo en la formación de docentes: Ejemplos con uso de tecnología interactiva", *Revista el Cálculo y su Enseñanza*, Vol. 5, pp 1-26

Gardner, H. (2014). *Inteligencias múltiples: la teoría en la práctica*. Buenos Aires, Argentina: Paidós Educador.

González, A. (2014) "La matemática nunca deja de ser un juego: investigaciones sobre los efectos del uso de juegos en la enseñanza de las matemáticas", *Revista Educación Matemática*, Vol. 26.

Hannaford, C. (2008). *Aprender moviendo el cuerpo: no todo el aprendizaje depende del cerebro*. Pax México.

Hannaford, C. (2011). *Cómo aprende tu cerebro*. Pax México.

Laski, E. V. and Siegler, R. S. (2014) *"Learning from Number Board Games: You Learn What You Encode*". American Psychological Association, Vol. 50, No. 3, pp 853-864.

Muñiz, L. (2014) "El uso de los juegos como recurso didáctico para la enseñanza y el aprendizaje de las matemáticas: estudio de una experiencia innovadora", *Revista Iberoamericana de Educación Matemática*,
No. 39, pp 19-33.

Rajkovic, A., Ruzic, M. & Ljujic, B. (2020) *"Board Games as Educational Media: Creating and Playing Board Games for Acquiring Knowledge of History"*. IARTEM E-Journal, Volume 11, Number 2.

Ramos, E. (2014). *Dominó con potencias*. Universidad de Chontalpa, Tabasco, México.

Simosa, H. (2009). *Ciencia y arte del dominó*. Caracas, Venezuela. Raúl Clemente Editores C.A.

Villanueva, A. (2014). *Dominó de ecuaciones*. Universidad de Guadalajara, México. Tesis de pregrado.

Zambrano, A. (2005) "Conocimiento, saber y pensamiento: una aproximación a la didáctica de la matemática". *Revista Equis Ángulo*, No.1, pp 1-6.