

Designing a Digital Game on Ratio and Proportion Concept: Integrating Environmental Awareness Through Contextualized Problem-solving

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Abstract: This study aimed to design a mathematical problem-based digital game focused on ratio and proportion, incorporating environmental awareness contexts. Using a design-based research methodology, four expert researchers and 24 middle school students participated in the evaluation, offering valuable feedback for refining the game. Data collected through observations and interviews were analyzed qualitatively using open and axial coding. The study's findings emphasize that effective digital game design integrates three key areas: foundational knowledge of digital gaming, informed by educational technology and game-based learning experts; mathematical theories and approaches, developed with input from mathematics educators; and practical, hands-on knowledge, gathered through student involvement. Additionally, the inclusion of entertainment elements and game mechanics ensured the balance between engagement and educational value. The results underscore the importance of interdisciplinary collaboration in creating educational digital games that are both pedagogically effective and engaging for students.

Keyword: Digital game, Design based, Problem solving, Game design

1. Introduction

Digital games have demonstrated numerous benefits in education, as supported by extensive research in the literature. They have been particularly effective in promoting conceptual understanding and affective development, especially in mathematics education (Ke, 2016). However, achieving educational success requires careful and effective game design. Games that lack a well-thought-out design process are unlikely to yield meaningful outcomes. Thus, it is crucial to examine the principles of effective game design in the context of mathematics teaching and learning.

1.1 Conceptual Framework

Designing educational digital games is a complex process that requires a structured approach (Toda et al., 2018). Various methodologies, such as ADDIE, Waterfall, and Mesh provide frameworks for this process, all of which involve multiple stages (Alkan et al., 2024). These stages often follow a cyclical structure, typically including design, implementation, and evaluation (Ke & Im, 2014). Each stage is dynamic and interdependent; for instance, encountering issues during implementation or evaluation necessitates revisiting and refining the design phase (Plomp, 2013). This iterative process continues until the game is fully developed and error-free (Gaydos, 2015).

An effective digital game should also prioritize user accessibility, ensuring simplicity and compatibility with diverse tools to maximize usability in educational contexts (King-Sears, 2009). This consideration is particularly relevant in developing countries like Turkey, where the use of educational digital games remains limited (Alkan & Ada, 2023). Combining multiple gameplay types can help digital games reach a broader audience, but the integration of game mechanics with educational objectives—especially in mathematics—is critical (Arnab et al., 2015). Properly designed mechanics, including context, difficulty, and gameplay, are essential for creating successful problem-based digital games (Ke, 2016).

Both the design aspect and the educational aspect of the digital game need to be designed effectively. In the context of mathematics education, incorporating approaches to mathematical learning, teaching, and thinking is crucial (Alkan & Ada, 2023). Problem-solving holds a central place in mathematics education, as extensively documented in the literature (Gurat, 2018). Importantly, the context in which mathematical problems are presented significantly impacts their effectiveness (Alkan et al., 2024). Moreover, it is more likely that using real-world contexts familiar to students increases engagement and success in problem-solving activities

(Freudenthal, 1984). In designing digital games for mathematics, it is essential to integrate mathematical knowledge and skills with purposeful game design. For digital games focused on problem-solving skills, particular attention should be given to defining mathematical problems and fostering effective problem-solving behaviors. Research highlights that a well-designed mathematical problem needs to be cognitively demanding, offer multiple solution pathways, relate to real-world situations, and promote collaboration (Takahashi, 2021). At this point, it is crucial to consider both digital game design and mathematical knowledge/skills with input from experts to ensure their pedagogical value in the context of digital games in mathematics education. Therefore, the aim of this study was to design a mathematical problem-based digital game related to the concept of ratio and proportion by integrating it into different contexts regarding environmental awareness.

2. Method

This study employs a design-based research (DBR) methodology, which allows for an in-depth examination of the design, research, and implementation processes using various methods (Wang & Hannafin, 2005). DBR is commonly used to investigate the design and effectiveness of educational materials (Borg & Gall, 1983).

2.1 Research Process

This study utilized a cyclic model inspired by the Kapp et. al. (2013) framework. The model comprises multiple stages, with each stage involving reviews by both users and trainers. Active user participation during the game design process is essential for ensuring the digital game's effectiveness and eliminating problems such as bugs, overly complex gameplay, or unclear tutorials (King-Sears, 2018). The stages of the cyclic model are outlined below:

2.1.1 Development stages of digital games

Stage 1. Pre-production. The primary goal of this stage is to define the objectives and content of the game. This begins with establishing the mathematical foundation of the game, which serves as its core structure. Following this, the game's mechanics, gameplay, and context are designed to align with the mathematical framework.

Stage 2. Production. This stage is divided into three phases: alpha, beta, and final version. In this stage, *the Alpha phase* introduces the basic context and mechanics of the game. *The Beta phase* represents a nearly complete version of the game, during which minor bugs are identified and addressed in this stage. Moreover, *the final version illustrated that* the game is fully developed, with all issues resolved in the context of this stage.

Stage 3. Game Release. In this final stage, the game is published and made ready for use. In this study, the production phase of the digital game was the primary focus. Therefore, the findings related to the pre-production and production stages were discussed in detail.

2.2 Participants

The digital game developed during the production phase of this study was tested with the target group. In the alpha stage, it was reviewed by three expert researchers with doctoral degrees in mathematics education, one specialist researcher with a doctorate in educational technology, and one specialist researcher with a doctorate in educational game studies. Following this stage, the game was piloted with 10 gifted 8th-grade students, whose feedback was used to identify technical errors and usability issues within the initial prototype. Based on this feedback, the researchers made several revisions to improve the game. The revised version was subsequently tested with 8 typical 7th-grade students to identify any remaining errors, followed by 6 typical 7th-grade students in the Beta phase, who evaluated the game's difficulty level, solution-strategy diversity, and collaborative dynamics. In total, four expert researchers and 24 students (10 gifted 8th-grade students and 14 typical 7th-grade students) participated across the sequential stages of the evaluation process, each contributing distinct, stage-specific insights for refining the digital game.

2.3 Data Collection Tools and Data Analysis

The primary data collection tools in this study were observations and semi-structured interviews. After the expert group played the digital game, semi-structured interviews were conducted, and necessary adjustments were made to the game based on their feedback by researchers. To ensure accurate data capture, the students' gameplay sessions were recorded using Movavi Screen Recorder. This allowed researchers to observe the challenges students faced while playing the game and identify in-game errors. As a result, the

researchers were able to address and resolve the issues encountered during the students' gameplay. The data obtained through observation and interview were analyzed with the open and axis coding approach and qualitative findings were revealed.

3. Results

3.1 Findings on the Pre-production stage: *Mathematical Teaching Objectives*

The findings related to the pre-production phase focus on key components, including the mathematical teaching objectives, context, gameplay, and mechanics of the digital game. Interviews with an expert team regarding mathematics educators revealed that a primary goal of the digital game should be to create a problem-solving environment for students. Based on this feedback, the researchers selected the concept of ratio and proportion as the central theme for fostering problem-solving skills. A thorough literature review further guided this decision, highlighting seminal works by Lamon et al. (1993) and Van Dooren et al. (2005), both of which emphasized problem types relevant to ratio and proportion. These studies identified four key problem types associated with the concept of ratio/proportion, which were incorporated into the game design. The types of problems used in this digital game are given below (Lamon et al., 1993):

Problem type 1 - Well-chunked measures. This type involves comparing two extensive quantities to derive an intensive measure, such as a rate (e.g., speed measured in miles per hour or price in dollars per item). These comparisons are often linked by a third, well-known quantity that connects the two measures.

Problem type 2 - Part-whole: This type focuses on measuring a subset of a whole by examining the relationships between its components. Examples include ratios like boys to girls in a class or correct to incorrect answers on a test.

Problem type 3 - Associated sets. In this type, the relationship between two sets or elements is ambiguous or undefined without the additional context provided in the problem. Examples include pairs like "people and pizza" or "aliens and spaceships," where explicit instructions are needed to establish meaningful ratios.

Problem Type 4 - Extenders and minimizers. This category involves proportional, continuous one-to-one mappings between two related quantities, often representing measurements like height, width, length, or circumference. It encompasses operations that extend (lengthen) or minimize (shorten) these measures, or relationships between two properties of the same element, such as length and width.

The overlap between the problems identified in both studies reinforced their relevance and applicability. Consequently, the digital game developed for this study integrated these four problem types to provide a robust and authentic learning experience centered on ratio and proportion.

4. Context

After identifying the problem types, an appropriate context was selected for the game. Research suggests that meaningful real-world contexts increase students' engagement in digital game-based learning (Kucher, 2021). Accordingly, two contrasting environmental contexts were designed: a green-environment context emphasizing sustainability through scenarios such as renewable energy, reforestation, and wastewater treatment, and an industrial-development context featuring thermal power plants, chemical production, and land conversion without prioritizing environmental conservation. To isolate the effect of contextual framing, all mathematical content, gameplay mechanics, artistic design, and numerical problem parameters were kept identical across the two versions of the game. Thus, the only difference between the two versions was the environmental narrative embedded within the mathematical problems.

4.1 Game Type, Gameplay, and Game Mechanics

The two contexts within the digital game are set in a city environment, where users are tasked with addressing various urban needs. Given the strategic and thematic focus of the game, it was categorized within the city-building or environment-based game genre. In this digital game, users were encouraged to engage in problem-solving scenarios by managing the construction of structures such as thermal power plants or solar panels, fostering both strategic thinking and contextual awareness. The gameplay aligns with the city-building and strategy game genre, featuring a bird's-eye view camera perspective. This design allows players to navigate and interact with the city environment dynamically, maintaining a strategic overview typical of games in this category. This design enabled the researchers to examine how different contextual narratives could be

incorporated into identical mathematical problems without altering the underlying mathematical structure or gameplay.

Findings on the production stage

4.1.1 Prototype

During the production phase, the researcher initially developed a prototype. A city environment and animated movements were designed as the foundation. Each level of the game was tailored to address a specific problem type, with two distinct contexts integrated into the game design. While the contexts varied, all other problem-related variables were kept consistent. For each context, the game was structured into four levels, corresponding to the four problem types. In the initial prototype, users were required to solve mathematical problems by typing their answers into designated fields during gameplay. However, a technology expert critiqued this approach, noting that the game relied too heavily on text and lacked essential gameplay elements, diminishing its appeal as a game. In response to this feedback, the researchers redesigned the game to incorporate greater flexibility and animation. The revised version allowed users to interact with the game by clicking buttons instead of typing. Players could now construct buildings and develop the city by purposefully clicking the relevant buttons as many times as needed to complete tasks. This modification enhanced the gameplay experience, making the game more engaging and interactive.

4.1.2 Challenges and adjustments in the Alpha phase

The alpha phase focuses on testing and refining the mechanics developed during the prototype phase, ensuring they function effectively and are free from bugs. At this stage, new mechanics may be added or removed, provided the core structure of the game remains unchanged. Within this sub-phase of the production process, adjustments were made to the prototype game and subsequently presented to students for evaluation. The evaluation consisted of two steps. First, the game was presented to a group of 10 students, and any identified errors were corrected. The revised game was then presented to a second group of 8 students for further testing. The primary purpose of this student gameplay was to identify and address in-game errors that might have been overlooked by the researchers. This iterative process revealed several issues within the two digital games across different contexts. For instance, some keys either failed to respond or did not produce the desired outcome when pressed. At the second level, pressing the button intended to restart the game instead caused students to exit the game entirely. Similarly, at the third level, typos were detected in the game script. Based on user feedback and observations, these errors were systematically corrected, including fixing the non-responsive keys and script typos. This iterative testing and revision process significantly enhanced the functionality and user experience of the digital games.

A key challenge identified during the alpha phase was students' initial difficulty in understanding what to do in the game. This issue was particularly pronounced among students unfamiliar with digital games, who struggled with camera navigation and identifying the game's objectives. In contrast, students who frequently played digital games reported no such difficulties. To address this, an information box was added to both game contexts, providing detailed guidance on-camera controls and game objectives for novice players. Following this addition, the revised games were tested again with students. Although the information box helped students understand the gameplay more quickly, some still required teacher assistance. The primary reason was their tendency to ignore the information box. To mitigate this, the game instructions were supplemented with visual aids alongside the written guidance. This adjustment enabled students to play the digital games independently without requiring teacher intervention.

Researchers modified the cognitive difficulty of the games based on student feedback. It was observed that the first level posed greater cognitive challenges compared to the second and third levels, leading to frustration and reduced motivation among some students. For instance, the problem-solving tasks in the first level were deemed more demanding. In response, the researchers reorganized the levels and adjusted the problem types to align with students' cognitive capacities and maintain their engagement (see Table 1).

Table 1: Change in game levels and problem types in the digital game

Game levels	Type of problem in the first case	Type of problem in the last case
Level 1	Well-chunked measures	Associated sets
Level 2	Part-whole	Part-whole
Level 3	Associated sets	Well-chunked measures

Game levels	Type of problem in the first case	Type of problem in the last case
Level 4	Extenders and minimizers	Extenders and minimizers

Another issue identified with the digital game was its brevity for gifted students, who completed all levels in approximately 20 minutes. Since one of the primary goals of the game was to observe changes in students' perceptions based on the context, additional levels featuring more challenging problem scenarios were incorporated to extend the playtime. Specifically, new problems were added to the digital games at the third and fourth levels, increasing the overall duration of gameplay to at least 30 minutes. Additionally, an educational gaming expert who tested the game suggested incorporating cutscenes to enhance its contextual effectiveness. Following this recommendation, context-driven cutscenes were added to the game. These cutscenes were presented to students, who provided positive feedback, indicating that the additions improved their overall experience and engagement with the game.

4.1.3 Beta phase: Refinement and educational evaluation

Following the resolution of foundational issues during the Alpha phase, the Beta phase commenced. At this stage, no new mechanics were introduced to the digital game, as it was nearing completion. The primary focus was on resolving in-game bugs and evaluating the educational aspects of the game. In alignment with Gurat's (2018) framework, which emphasizes that effective problem-solving tasks should balance difficulty, promote cooperation, and allow for diverse solution strategies, specific measures were implemented. Six seventh-grade students were purposefully selected to identify remaining usability issues and refine the educational design through iterative feedback. During this phase, the digital game was tested with these six students, divided into groups of two to foster collaboration. The outcomes of the Beta phase were analyzed in terms of problem difficulty, the variety of problem-solving strategies employed, and the level of cooperation achieved among students.

It was observed that comprehending the problem scenario and devising appropriate solutions posed significant challenges. However, feedback from students indicated that this level of difficulty did not diminish their motivation to engage with the game. Instead, the challenge appeared to encourage persistence and active problem-solving. Using different problem-solving strategies is a crucial skill in the mathematics teaching and learning process. In this context, it was deemed beneficial to encourage users to apply various solution strategies to the problems within the digital game. Specifically, the difficulty mechanics of the game were designed to prompt students to experiment with different strategies. It was observed that students sometimes employed non-mathematical strategies, such as guessing illogically or selecting the most expensive solar energy option, which lacked a data-driven basis. However, as students made mistakes within the game, they were able to adjust their strategies in a more effective manner. Thus, the digital game provided students with an opportunity to explore multiple problem-solving approaches related to the concept of ratio and proportion. This process allowed for the development of a deeper understanding of the ratio and proportion concepts, and thinking desirable through trial, error, and strategic adaptation.

In terms of collaboration, the Beta phase of the digital game involved students working in pairs. This setup allowed them to discuss their findings, deliberate on which device or building to choose, and justify their decisions. The structure of the game's scenario and the nature of the problems were designed to encourage collaboration. The key factor driving this cooperation was the cognitive challenge posed by the game. Given the difficulty of the tasks, students were required to work together to solve the problems. When considering the game's difficulty, solution strategies, and collaborative aspects, it can be concluded that this digital game is an effective mathematical activity, fostering the development of problem-solving skills through teamwork.

The final version of the digital game

4.1.4 The problem with the type of associated sets in the first level of the digital game

As illustrated in Figures 3a and 3b, the context is the only variable that differs between the two problems, while all other elements remain consistent across the games.

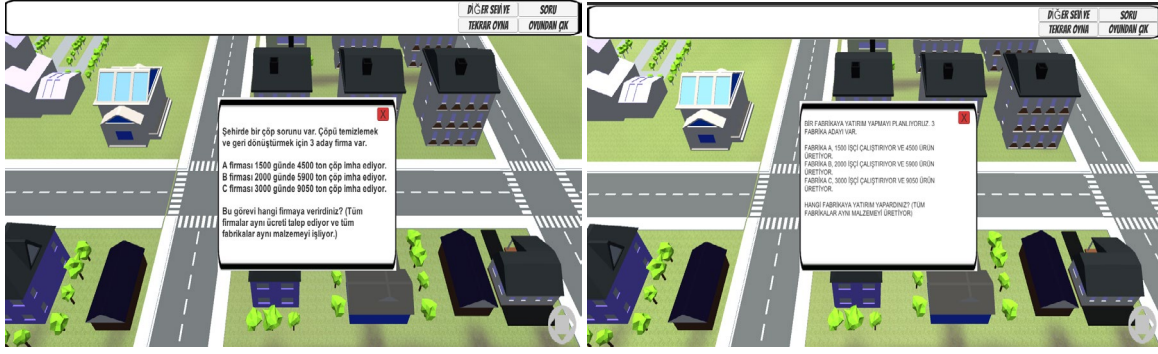


Figure 3a and Figure 3b: Game visuals for the associated sets problem type in both the green environmental context and the unregulated industrial development context

Industrial-development context. Students solved an associated-sets ratio problem by comparing the productivity of three factories using worker-to-output ratios to determine the most efficient investment.

Green-environment context. The same mathematical problem was presented within a waste-management scenario, where students compared the efficiency of three recycling companies using identical numerical data to select the most efficient provider.

4.1.5 The problem with the type of part-whole in the second level of the digital game

As shown in Figures 4a and 4b, the two versions differed only in their contextual framing, while all mathematical and gameplay elements remained identical. The industrial-development context depicted environmental degradation, whereas the green-environment context illustrated ecological restoration after level completion.



Figure 4a and Figure 4b: Game visuals for the part-whole problem type in both the green environmental context and the unregulated industrial development context

Industrial-development context: Students solved a part-whole ratio problem by comparing the production efficiency of three chemical factories to identify the facility with the highest output per unit of time.

Green-environment context: The identical mathematical problem was presented within a wastewater treatment scenario, where students compared the treatment efficiency of three facilities using the same numerical values to determine the most effective option.

4.1.6 The problem with the type of well-chunked in the third level of the digital game

As illustrated in Figures 5a and 5b, the two versions differed only in their contextual framing, while all mathematical and gameplay elements remained identical. The industrial-development context depicted environmental degradation, whereas the green-environment context emphasized renewable energy and ecological sustainability.

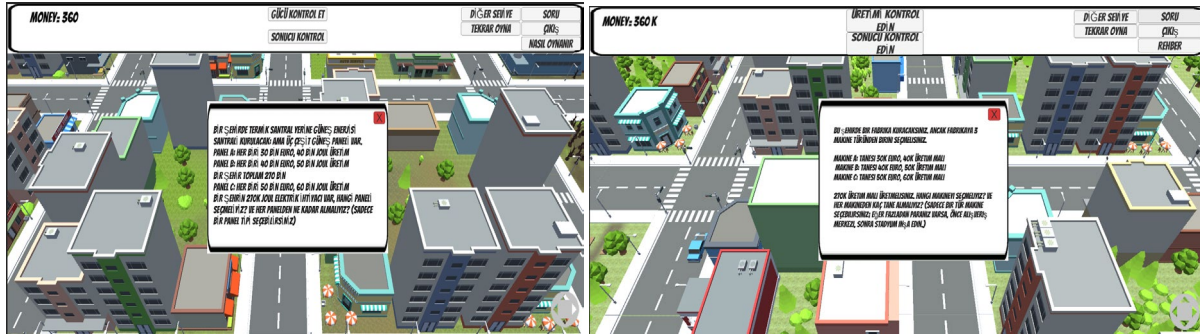


Figure 5a and Figure 5b: Game visuals for the well-chunked problem type in both the green environmental context and the unregulated industrial development context

Industrial-development context: Students compared the cost and production capacity of three machine types to identify the most cost-effective solution for achieving a predetermined production goal.

Green-environment context: The identical mathematical structure was presented within a renewable energy scenario, where students compared the cost and energy output of three solar panel types to determine the most cost-effective option for meeting the city's electricity demand.

4.1.7 The problem with the type of extenders and minimizers in the fourth level of the digital game

As illustrated in Figures 6a and 6b, the two versions differed only in their contextual framing, while all mathematical and gameplay elements remained identical. The industrial-development context depicted environmental degradation through deforestation, whereas the green-environment context emphasized reforestation and ecological restoration.

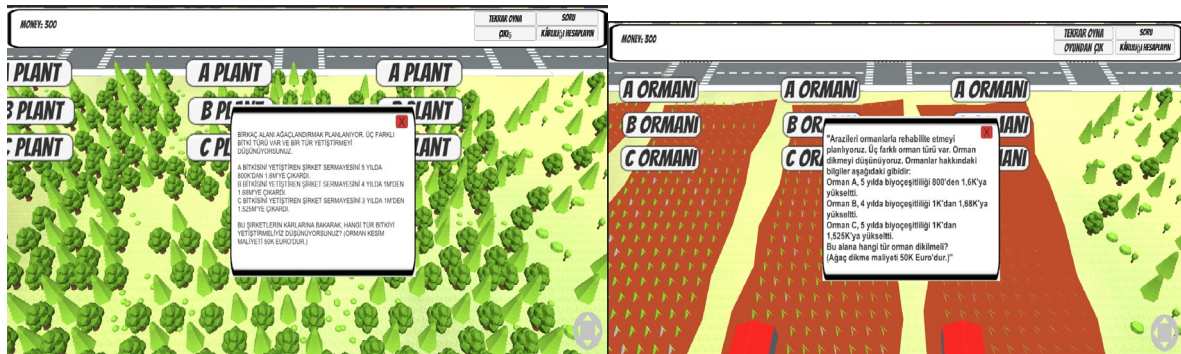


Figure 6a and Figure 6b: Game visuals for the extenders and minimizers problem type in both the green environmental context and the unregulated industrial development context

Industrial-development context: Students solved an extenders-and-minimizers ratio problem by comparing the profitability of three agricultural investment options to determine the most advantageous alternative following land conversion.

Green-environment context: Using identical numerical parameters, students solved the same mathematical problem within a reforestation scenario by comparing the profitability of three tree species to identify the most suitable option for ecological restoration.

5. Discussion and Conclusion

The findings should be interpreted within the context of a design-based research methodology. The study aimed to iteratively improve the digital game through expert and student feedback rather than to establish broadly generalizable effectiveness claims. Future research should evaluate the proposed design principles with larger and more diverse student populations and investigate whether similar outcomes emerge across different contextual themes. The key finding of this study is the critical role of involving the target group (i.e. middle school students in this study) and an expert team at various stages of the digital game design process (Silva, 2019). Feedback from these groups allowed researchers to identify and address in-game and other issues (i.e., bugs) that might have gone unnoticed otherwise. By incorporating this feedback, the researchers were able to implement necessary corrections and improvements.

Incorporating mathematical theories and approaches, together with feedback from students and domain experts, is essential for ensuring the mathematical quality of educational digital games. Consistent with previous research (Alkan & Ada, 2023; Lodge et al., 2018), the findings suggest that effective game design should include a variety of problem types and an appropriately calibrated level of cognitive difficulty. In the present study, these design features encouraged students to explore multiple solution strategies, engage in trial-and-error reasoning, and collaborate during problem solving. These findings support previous research indicating that well-designed cognitive difficulty mechanics can stimulate mathematical thinking, discussion, and collaborative learning (Fadda et al., 2021; Wang & Huang, 2021).

The findings of this study suggest that effective digital game design requires integrating three key areas of knowledge. The first is foundational knowledge of digital gaming, informed by contributions from experts in educational technologies and digital game-based learning. The second is mathematical theories and approaches, developed with input from mathematics educators. The third is practical, hands-on knowledge, shaped through the involvement and feedback of students during the design process. Moreover, the inclusion of entertainment elements, game mechanics, and educational content within the digital game ensured a balance between engagement and pedagogical effectiveness. The findings of this study highlight the importance of interdisciplinary collaboration in designing effective educational digital games, integrating insights from educational technology, mathematics education, and student experiences. This approach not only enhances the cognitive and pedagogical value of the game but also ensures its accessibility and engagement for the target group. Future research could further explore the role of game difficulty mechanics in fostering problem-solving skills, mathematical reasoning, and collaborative learning. Additionally, longitudinal studies could investigate the long-term impact of digital games on students' cognitive development and motivation in various educational contexts. Expanding this research to include diverse age groups, cultural settings, and mathematical concepts would provide a broader understanding of how digital games can effectively support learning outcomes across disciplines.

6. Limitations

This study has two main limitations. First, the relatively small sample size (24 students) limits the generalizability of the findings. Second, different student groups participated in successive design iterations; therefore, the findings should be interpreted as supporting the iterative refinement of the game rather than as directly comparable results. Future studies should validate the proposed design principles with larger samples and a single participant group across all design stages.

Data Availability Statement: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Conflict of Interest: The authors declare that they have no conflict of interest.

Ethical Considerations and Informed Consent: Participation in the study was voluntary, and participants were informed that they could withdraw from the study at any time without any negative consequences. Written informed consent was obtained from the parents or legal guardians of the students prior to participation, and assent was obtained from the students. All data were collected anonymously and used solely for research purposes.

Informed Consent: "Informed consent was obtained from all individual participants included in the study."

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