Treasure Hunt as a Method of Learning Mathematics

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Abstract: Future jobs will require students to handle interdisciplinary knowledge that goes beyond computing techniques when it comes to mathematics education. The traditional approach to education insufficiently develops the needed mathematical skills and mathematical literacy and there is a need to devise new methods of teaching and learning. We introduce game-based learning in mathematics education through carefully designed activities aiming at interdisciplinarity and increasing students' motivation, mathematical and digital literacy, physical activity, level of knowledge, etc. We propose the treasure hunt cantered around cryptography as an example activity that satisfies the described criteria. Parts of the clues leading to the treasure are hidden using different ciphers, following the historical development of cryptography. The treasure hunt is time-limited to one hour and, to successfully complete the search, students need to learn, adapt, and apply different methods of encryption and decryption. To complete the hunt in time, students must work as a team and effectively divide the tasks among themselves. Students have at their disposal all instructions and manuals in written form, and they are compelled to read with comprehension. We conducted the activity with several groups of high school students with no previous knowledge and skills in the field of cryptography. Students were asked to take a pre-test and post-test survey about their experience and familiarity with cryptography concepts. Most teams completed the search and achieved all learning outcomes. In the paper, we will describe the activity in detail, analyse the test results and give additional examples of game-based activities in mathematics education.

Keywords: game-based teaching, teaching of mathematics, treasure hunt, cryptography, literacy in STEM, digital literacy

1. Introduction

Future jobs will require students to handle interdisciplinary knowledge that goes beyond computing techniques when it comes to teaching mathematics (Staats, S. (2014)). These jobs will require lifelong learning, teamwork and applying what they learn to new situations. Most importantly, they will need to make decisions based on a quick but effective analysis of the situation and the choice of appropriate cost-effective strategies to address potential risks. Moreover, digital literacy is crucial for learning, life and work (European Commission (2016)).

Mathematics is the universal language and the foundation of education. Analytical mathematical thinking is indispensable and enables real-life problem solving in various disciplines (Haylock and Thangata (2007)).

Mathematics as a universal language describes nature, our environment and the processes that affect our daily lives. Therefore, mathematics becomes important not only for those who want to become mathematicians, mathematics teachers, physicists, engineers, etc., but it is also needed in many other fields and professions. It is no longer enough to just require students to learn mathematical content that is an integral part of the curriculum. Students are expected to acquire mathematical precision, critical and analytical thinking and reasoning, which are inherent in mathematics and are developed through mathematics education.

All this points to the need to develop students' mathematical literacy. Mathematical literacy (OECD (2022)) includes an individual's ability to recognize and understand the role that mathematics plays in the world, make well-founded decisions, and apply mathematics to everyday life needs.

Today's education faces many challenges. The traditional approach to education does not sufficiently develop the mathematical skills and mathematical literacy, therefore new methods of learning and teaching need to be developed. To develop the skills needed today, mathematics teachers should use digital technology and cross-curricular teaching in the classroom (Volk et al. (2017), Chai (2019)).

Teaching methods should put students in situations similar to the situations they will face in the future. These situations are unfamiliar and require them to test what they have learned and adapt familiar procedures and methods. To keep focus and motivation high, students must be independent and ready to adopt new concepts and procedures in a short time. To ensure effective teaching, the teacher should identify a threshold concept (i.e. a concept without which a learner cannot progress) and focus attention on the method of introducing and adopting the concept (Meyer and Land (2003)). These threshold concepts should be part of cross-curricular topics to provide students with a fresh perspective and reinforce critical content (Kurisma and Ratinen (2021)).

With the above guidelines in mind, we present methods of game-based teaching to achieve the described goals in mathematics and other subjects.

1.1 Game-based teaching and learning

Games have been a part of the educational process since ancient times (Vankúš (2005)). In the Republic and Lows, Plato advocated the use of games in education because this is how students prepare for their future jobs. In ancient Rome, the first schools were called "ludi", meaning "games", and the aim of using games was the physical development of students.

From an early age, children learn by playing. This very effective form of teaching can be integrated into formal education. There are numerous papers describing the characteristics of game-based learning and teaching, the advantages and disadvantages, and the challenges that the teacher encounters in this type of teaching. Game-based learning and teaching uses the motivation and competition that naturally arise in play to achieve the goals and outcomes of teaching. There are numerous studies showing many positive effects of such learning (Ku et al. (2014), Vankúš (2021)) and although some of the articles report mixed results, none report a negative impact. An extensive literature review of serious games based on multiple criteria (Backlund and Hendrix (2013)) found an overwhelming amount of evidence that serious games have a positive impact on learning and motivation. The impact of serious games in mathematics education shows a significant improvement in mathematical skills in the different grades and class groups (Hieftje et al. (2017), Fraga-Varela, Vila-Couñago and Martínez-Piñeiro (2021)).

Teaching through play is certainly one of the approaches we use to develop students' mathematical literacy and create learning situations that resemble everyday situations. Students need to learn quickly and apply what they have learned, make quick decisions and devise strategies for action. The most commonly used tools for game-based learning of mathematics are computer games and board games. We distinguish between games that promote general mathematical thinking or reasoning (e.g. Chess, Trills, Sudoku, etc.) and games where mathematical tasks can be adapted to very specific domains and set for progress in the game (snakes and ladders, gallows, escape rooms, etc.).

There are examples of the implementation of game-based learning in mathematics education that take a similar approach to the one presented in this paper (Deeb and Hickey (2019), Seebauer, Jahn and Mottok (2020)), all of which report very positive results.

The hunt described in this paper was developed as part of the InAMath - An interdisciplinary approach to mathematical education project (https://inamath.uniri.hr/), a project that aims to develop and implement interdisciplinary activities in primary schools.

Now we give an interesting example of game-based learning in mathematics education. Learning definitions with understanding and operational knowledge of definitions is a challenge for students. Game-based learning is a great way to help them overcome this challenge. We suggest the game Mathematical Alias to learn definitions and apply them directly in the game. The game is easy to prepare. The teacher prepares cards with concepts that relate to a specific mathematical content. For example, if the content is geometric solids, the teacher prepares cards with concepts such as solid, cube, pyramid, side, edge, vertex, ... For younger primary students, the teacher can prepare images of geometric objects or 3D models of objects to complement the cards. A student scores points in the game if he/she explains the terms on the card to his/her partner without naming the term or similar words, or if he/she guesses the term based on his/her partner's explanation.

The cards with terms that a teacher can use in Mathematical Alias for younger primary school students can be found at https://inamath.uniri.hr/mathematical-alias/.

It is well known that using ICT as a cognitive tool in the classroom can improve educational outcomes (Jonassen and Reeves (1996)). Familiarising oneself with a programme interface can be boring for students and treasure hunt can be used to overcome this. SURFER is a free programme developed as part of the project IMAGINARY (imaginary.org) in cooperation with the Mathematisches Forschungsinstitut Oberwolfach and the Martin Luther

University Halle-Wittenberg. SURFER is a powerful cognitive tool that can be used in mathematics lessons. With SURFER, one can draw with formulas and investigate how the algebraic change of a formula infects the shape of a 3-dimensional object given with this formula.

We give an example of a hunt through which students can familiarise themselves with the SURFER interface.

- Clue1: Open SURFER, find a fruit and read the number that starts the fruit equation. Memorise this number!
- Clue2: Find one among the Fantasy Surfaces whose ordinal number is ten times greater than the number you discovered in clue 1. Do you recognise the shape? What is the value of the number for the object shown? Memorise this number!
- Clue3: Among the World Record Surfaces, find the one that looks like it is composed of two birthday caps. Set the maximum value of the number *a* so that the two birthday hats merge? Remember this number!
- Clue4: In SURFER you can look at every 3D object from all sides. Find an object that always looks the same from whichever side you look at it! Have you found it? What does it look like? What is it called?
- Clue5: Look at the equation of the object you discovered in clue 4. Replace all the twos with fours? What does the resulting object remind you of now?
- Clue6: Do you like games of chance? Find a mathematical object that is often used in casinos. What are the exponents of the variables *x*, *y*, *z*? Memorise this number!
- Clue7: Is there a universal symbol of love hidden among the drawn mathematical objects? Have you found it? What colour is it?
- Clue8: "________ on the fire Feathers on my breath". Do you recognise the song by Massive Attack? Which word is missing? Do you see an image somewhere that reminds you of the missing word? What colour is it?
- If you answer all the questions correctly, the answers will help you draw the treasure. Draw a mathematical surface that consists of two objects (remember: we can draw two objects by multiplying their equations). One part is the shape you discovered in clue 4, let a^2 be the number you discovered in clue 6 and colour it in the colour you discovered in clue 7. The other part should be the object you discovered in clue 5, in the colour you discovered in clue 8. Set the zoom to the number you get when you subtract the numbers you discovered in clues 2 and 3 from the number in clue 1. What do you see? Now set the zoom to the number from clue 2. What do you see now? Play around a bit with the zoom slider.
- What do you get? Describe the treasure you found!

For more examples of using games in the primary classroom, see Holenko Dlab et al (2020).

2. Treasure Hunt as a Method in Mathematics education

We will pay special attention to treasure hunts as a teaching method, with a focus on mathematics education. Treasure hunts are widely used in the gamification of mathematics teaching, but the games are usually used for repetition of what has been learned. In this paper, we explain how to use a treasure hunt as a method for teaching mathematics. A carefully designed treasure hunt can create a stimulating learning environment, but it can also have several other positive effects. Treasure hunts are often organised as a team effort and thus improve students' social skills, especially collaborative learning and problem solving. It is also very easy to design a treasure hunt in such a way that it involves the physical activity of the students, if the educational environment allows for it. This has many advantages: it encourages mental activity and improves concentration and motivation. Using physical activity as often as possible in the classroom addresses the major problem of insufficient physical activity among today's children and youth (Baran et al. (2020)). Another major issue of today is digital literacy. Although people are taught from a young age how to use computers, mobile phones, tablets, etc., they often lack digital skills (van Laar et al. (2019)) in the sense that they do not know how to use the tool of information and communication technology to solve the challenges they face. The treasure hunt will be more interesting for the students and therefore it will be easier for them to achieve the intended results if they use several different methods and digital tools.

Especially in the lower primary grades, it is possible to design a treasure hunt that includes tasks from different subjects (mathematics, science, language and foreign language, physical education, computer science). We have designed and conducted such a treasure hunt with second grade primary school students and the students' reactions were more than positive.

When a teacher introduces treasure hunts into the mathematics classroom, he or she almost inevitably applies the principles of good practise, i.e. fostering reciprocity and cooperation among students, encouraging active learning, providing immediate feedback, emphasising time on task, communicating high expectations, and respecting different talents and ways of learning (Chickering and Gamson (1991)). Students are active, engaged, interested and motivated to acquire knowledge and solve problems. The treasure hunt is a way for students to have fun and relax, it is a way for students to repeat what they have learned, but it is also a very good way to learn mathematics. The treasure hunt is also a good way to make mathematics more popular.

Moreover, the aim of the modern education system is to create, as much as possible, an educational environment in which the student is an active participant in the educational process, while the role of the teacher is more of a mentor. In addition, the new curriculum advocates an increasing collaboration between teachers and the inclusion of an interdisciplinary approach in the teaching process. There is a great need to develop activities to achieve this. The treasure hunt is certainly one of the ways to achieve interdisciplinarity. According to Bloom's Taxonomy (Bloom (1956)), there are six levels of cognitive skills: Remembering, Understanding, Applying, Analysing, Evaluating and Creating. The goal is for students to reach the level of creativity. In the following discussion, we will illustrate how to reach a higher level of taxonomy using a treasure hunt as a teaching method.

There are many challenges in designing a treasure hunt with the aim of teaching mathematics. First, a topic must be chosen that is new but not too challenging so that students can master the content. The next challenge is to present the content, which can be done either orally by the teacher or through a carefully crafted text that is available to the students throughout the treasure hunt. The written text will help to solve a major problem of today's youth, namely reading illiteracy (OECD (2022)).

The most challenging part of the preparation is the design of the tasks. It is desirable that the tasks are interesting, that they are not too easy, because otherwise they will not achieve the desired learning effect, but they must not be too challenging either, because then most students might give up solving them. If the quest is a way to teach new content in regular classes, then it is desirable that all students actively participate in it.

2.1 Secret Hunt

2.1.1 Goals

The goal of the activity is to familiarise students with the history of cryptography and the different types of encryption and to develop collaborative learning, creativity, critical thinking, the ability to cope with new situations and the independent adoption of new procedures with the aim of direct application in solving new problems, ...

2.1.2 Learning outcomes

Upon completion of the activity, students will be able to define cryptography and describe a cryptosystem based on secret key encryption, describe and distinguish encryption and decryption procedures in cryptosystems based on the Caesar's and Vigerener's ciphers, apply an attack to cryptosystems based on the Caesar cipher, apply a decryption procedure to a simple Enigma machine, use Micro:bit and mBot.

2.1.3 Description of the activity

The main topic of the hunt is cryptography, the main goal of which is to ensure secret communication between two people, commonly called Alice and Bob, in such a way that no other intruder, commonly known as Oscar, can read their messages. Encryption is the process of converting plaintext into ciphertext using a key. The reverse process, decryption, allows you to convert the encrypted text into plaintext if you have the key. The scheme of the cryptosystem is shown in Figure 1. For more information about the cryptography see Stinson (2006).

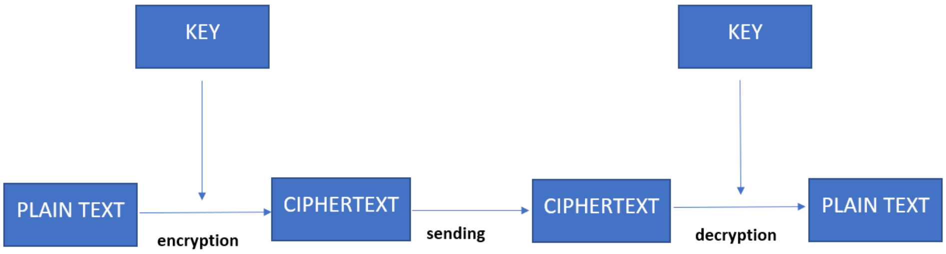

Figure 1: Cryptosystem

The hunt is designed for about 20 students, who are divided into four groups. Each group will have its own desk with all the necessary materials and a booklet with tasks and hints. The materials we have prepared for each group are:

- an Enigma machine, as shown in Figure 2,
- a set of visual cryptography transparencies (which reveal the image only when they overlap) with the following images: a flower with six petals, a happy smiley, a black circle, a 6-pointed star, the number 9 and a black square,
- a set of 5 Micro:bits prepared as follows: 4 "fake" Micro:bits which have been given a fake code (randomly decodes each letter into L, S or R) and on which 4 different stickers have to be stuck: flower with 5 petals, empty circle, a five-pointed star, a sad smiley,
- and a "real" Micro:bit that can be used as a decryption device with a black square sticker (identical to the one on the transparencies).

Figure 2: Enigma machine

At the beginning of the activity, all the devices needed for a successful hunt were displayed on the presentation table (Enigma, visual cryptography transparencies whose overlap reveals an image, two Micro:bits: one that serves as an encryption device and one that serves as a decryption device, using public key cryptography), as shown in Figure 3. Each device is accompanied by a short text containing a few sentences of general information about the device and the description of the corresponding cipher, including instructions for encryption and decryption with the obtained devices. Before the students started solving the puzzle, the team captains familiarised themselves with a short text about cryptography, its goals and uses, as well as all the devices on the presentation table and the instructions and encryption and decryption methods needed to solve the hunt.

The instructions on the presentation tables contain basic information about the Caesar cipher, the Enigma machine (definition of the key, an encryption method, a decryption method, an example), visual cryptography, basic information about public key cryptography and a brief guide to Micro:bit as an encryption or decryption device for public key cryptography.

Figure 3: Presentation desk

Each group of students is provided with a booklet containing the following clues.

- CLUE 1: Caesar encrypted an alphabet letter by replacing it with the letter 3 positions down in the alphabet. Guess a number less than 10 and discover a secret message: "YTRNRFLNSJIYMWJJSZRGJWXQCCQQC"
- EXPLANATION OF CLUE 1: This clue results in a decrypted text "TomImaginedThreeNumbersLXXLLX". The original text is encrypted using the Caesar cipher with key 5 (each letter is shifted 5 steps in alphabetical order) so that the encryption function is $e(x)=x+5 \bmod 26$, where x is the position of the letter to be encrypted, starting with position 0 for letter A. The decryption function is $d(x)=x-5 \bmod 26$, where x is the position of the letter to be decrypted.
- CLUE 2: Listen carefully to what Caesar says, lock the Enigma, and it will tell you what to do next: "OVDIDZPYOSDCYIUHTBDK"
- EXPLANATION OF CLUE 2: Students must figure out a key for the Enigma device to decrypt the message given in clue 2. As written in instructions for Enigma, key consists of a three-letter word (TOM) and the sequence of gears numbered 50, 60, 70. The previous hint gives the order of the gears: LXXLLX, i.e. 70-50-60 in Roman numerals. After decrypting the cipher text in clue 2, students discover the plain text: "TheSecretKeyIsTheNumber".
- CLUE 3: Put the transparencies together to figure out the secret key.
- EXPLANATION OF CLUE 3: By overlapping the transparencies and finding corresponding pairs, students get the image of a flower with six petals, a happy smiley face, a black circle, a six-pointed star, a number 9, and a black square.
- CLUE 4: Transparencies help you choose the right Micro:bit. Unlock it and discover the robot's path to the treasure: "EBEJBEE".
- EXPLANATION OF CLUE 4: Students must match the images on the transparencies and images on the Micro:bits. Only one image has a match: a black square. The message from Clue 2 is: the secret key is a number that, when combined with transparencies from the previous clue, gives a key to the Micro:bit, a number 9. Micro:bit has two buttons: A and B. On button B, the student selects a number (key): from 1 to 26. With button A, the student selects a letter to decrypt. By pressing A and B simultaneously, Micro:bit decrypts the selected letter with the selected key. Students decrypt the encrypted text of clue 4 with the Micro:bit and get the plaintext: "SRSLRSS".
- CLUE 5: Run me and follow the path, and when the path disappears, listen to what the Micro:bit tells you (L: left, R: right, S: straight).
- EXPLANATION OF CLUE 5: Students come to the road map where the track and the mBot are prepared. The earlier clue tells them the direction to go. Following the instructions of the sequence they discovered, students should reach the target field with the message "Good job!". The message must be illuminated with a UV lamp to reveal its meaning. If students made a mistake, they should go back to the spot and find the mistake and/or start over.

All the materials one needs to implement this hunt are available at: https://inamath.uniri.hr/treasure-hunt/.

3. Methodology

This was a case study conducted with high-school students with no prior knowledge of the topic. There was no further discussion of the concepts during the lesson. We were able to conduct the activity with elementary school students, but they needed some assistance to finish the game, so we decided not to do the survey with them.

All high school students were asked to complete a questionnaire before the game about their knowledge of cryptographic concepts used in the game. We gave them a brief overview of how the game worked and then asked them to play the game, read the instructions, and learn new concepts as they progressed through the game. The game is limited to 60 minutes.

After the game, we asked participants to complete a post-test about their experience with the game. We also asked them several questions to test their understanding and practical application of the concepts and algorithms that the game was designed to teach. The goal of this survey was to measure prior knowledge, attitudes, and new knowledge and skills acquired through the proposed treasure hunt.

This secret hunt was conducted for three years with over 200 students in different settings and as an extracurricular activity. The presented survey was conducted with 64 students on a single day. The students were participants in the Open Day of the Faculty of Mathematics in year 2022.

4. Results

The survey was administered to 18 groups of students, of which 16 groups finished the hunt. Of the 64 surveys, only 52 were completed in full and only those were considered. The survey consists of 3 parts. The first part consists of 5 questions and was conducted prior to the activity (pre-test) to test students' knowledge and general familiarity with the topic.

Table 1: Students' knowledge before the activity

	Yes	No
Do you know what cryptography is about?	32	20
Have you ever heard of Caesar's cipher?	30	22
Do you know what Caesar's cipher is?	11	41
Have you ever heard of an Enigma?	48	4
Do you know how an Enigma works?	11	41

The second part consists of 2 questions and was administered after the activity to assess student attitudes.

Table 2: Student attitudes after the activity

	Yes	No
Was the activity interesting for you?	50	2
Did you learn anything new?	49	3

The third part was also conducted after the activity and includes 4 open-ended questions designed to test students' knowledge of the activity topic. Participants' answers were scored 0 for a wrong answer, 0.5 for a partially correct answer and 1 for a completely correct answer.

In the following table we report the results of the test.

Table 3: Practical demonstration of knowledge after the activity

	Level of knowledge (score)		
	Poor (0)	Satisfactory (0.5)	Excellent (1)
What is the purpose of cryptography?	16	3	33
What must two people agree on if they want to use encryption in their communication?	1	29	22
Encrypt the word “RIJEKA” with Caesar's cipher using the key 4.	5	20	27
Describe the key for the Enigma machine.	20	3	29

The average score for the third part of the survey is 0.66, which can be considered more than satisfactory.

5. Discussion

Serious games are increasingly used to teach topics that are not related to the curriculum but revolve around an interesting or important problem. Cryptography is not part of the high school mathematics curriculum, which makes this topic ideal for this survey. Many students indicate in Table 1 that they have heard of Caesar's Cipher and Enigma, but very few indicated that they knew how these systems worked before the game.

The results of the practical section in Table 3 of the survey confirm the effectiveness of the proposed model. Students showed a measurable increase in their familiarity with the terminology and ability to apply the knowledge. For example, the vast majority of students were able to encrypt the word using the Caesar cipher, but some of them shifted the letters in the opposite direction and confused the encryption and decryption procedures. Most of the student teams managed to solve the game in time without any help or interaction with the teacher, and the average test score was better than expected. We should emphasise that students had no incentive (in the form of a grade or credit) to achieve a good score on the test, which is a compelling argument for game-based learning.

The positive feedback from the participants after the treasure hunt, which you can see in Table 2, is overwhelming. They expressed their satisfaction with the topics covered and the knowledge and skills acquired.

6. Conclusions and plans for future work

The aim of this paper was to present an interesting format for teaching mathematics that allows us to introduce new concepts to high school students through a fast-paced game that allows students to learn and apply mathematics and physically interact with mechanical and digital versions of algorithms and systems. This game encourages players to try different strategies, cooperate and discover new content on their own and finally test their skills.

The survey results show a very positive response to a learning format that leads to high motivation to learn new concepts. The practical part of the survey confirmed the effectiveness of the proposed model, where students showed an impressive, measurable increase in knowledge of a complex new topic. This approach can also be applied to other areas of mathematics with different levels of difficulty.

An interesting extension of this treasure hunt would be to create an online 3D multiplayer version of the hunt where students learn, test themselves and compete with others. This would probably increase the number of players in the game but would require a thoughtful and possibly significant revision of the activities.

It would be interesting to conduct a study to investigate the impact of the proposed method on the adoption of threshold concepts and the acquisition of higher order learning outcomes. The study should identify threshold concepts in first grade high school mathematics and include an experimental and a control group of high school students. The treasure hunt should be designed for the identified concepts and conducted with an experimental group while the control group is taught in a traditional way.

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