Formative Assessment in Digital Classrooms: Comparison through Bar Model Representations

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Abstract: Digital assessment introduces fresh possibilities for evaluating mathematics learning when students' work is created in digital settings. Teachers can modify task structures for remediating students' conceptual deficiencies and align the scope of assessed abilities and skills according to their learning needs. Two key processes underpin effective formative assessment: collecting data on student digital interactions and devising strategies to enhance performance. Among the many contributions of technology to contemporary pedagogy is its capacity to furnish students with an extensive suite of digital tools for exploration, practice, and representation. One such tool is the bar model virtual manipulative. The bar model, a core feature of the model method used in the Singapore mathematics curriculum, is a structured visual representation designed to support students' understanding of mathematical relationships in problem solving. In line with Bruner's Enactive–Iconic–Symbolic framework, the bar model occupies the iconic stage of learning, acting as a bridge between hands-on manipulation of objects and abstract symbolic reasoning. By using rectangular bars to represent known and unknown quantities, students are guided to visualise part-whole and comparison structures, comprising two key relational models that underpin many arithmetic and algebraic problems. This visual representation is particularly helpful for students who struggle with abstract concepts in word problems, as it allows them to explore mathematical representations dynamically. Moreover, research has shown that instructional strategy involving the comparison of visual representations effectively supports mathematics learning. A pilot study was conducted to explore the strategy's effectiveness with nine Grade 6 students, employing screen recordings and post-task group discussion as data collection instruments to capture student interactions and fractional reasoning while comparing bar models. Comparison of bar models fosters active learning through interaction with these representations, providing insights into students' mathematical thinking in the process. **The study highlights the role of visual modelling and scaffolding to promote** student-centred practices, fostering conceptual learning and relational reasoning in mathematics within technology-enhanced settings.

Keywords: Formative Assessment, Model Method, Comparison, Visual Representations, Student Dialogue

1. Introduction

Technology has played a key role in shifting classroom practices from traditional approaches centred on teacher demonstration and student practice to learning environments that promote students-driven engagement in the modelling process. Students can now actively engage in constructing mathematical models, with teachers stepping in to support learning through timely discussion and reflection. As international studies reveal a growing number of innovative practices (OECD, 2016), the integration of technology into mathematics classrooms continues to raise important pedagogical questions, particularly concerning how it reshapes teacher-student interactions, the role of representation, and students' mathematical reasoning. Rather than serving solely as instructional delivery mechanisms, digital tools create opportunities for students to demonstrate knowledge and errors in reasoning, receive timely feedback, and revise their thinking. Viewed through a formative assessment lens, using digital tools such as bar models on interactive platforms can support ongoing diagnosis of student understanding, misconceptions, and strategies, and enable more responsive instruction, where teachers can observe students' reasoning as it unfolds (Drijvers et al., 2016). Research has established that comparison effectively enhances mathematics learning as an instructional strategy (Rittle-Johnson et al., 2017). Building on evidence of its effectiveness, a programme was piloted at a Czech elementary school with nine Grade 6 participants who solved fraction-based word problems utilising a web-based bar model app on their mobile devices. The methodological approach involved collecting data from students' interactive activities while using the app, analysing their contributions during bar model comparison discussions, and interpreting patterns through qualitative analysis. Following this research design, the study investigates how comparing students' digital representations exposes misconceptions and helps teachers target conceptual gaps.

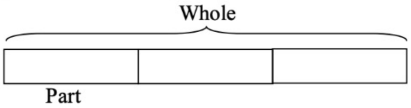
2. Theoretical Framework

2.1 The Model Method

Inspired by Greeno's part-whole and comparison schemas (Nesher et al., 1982; Kintsch & Greeno, 1985), the model method is a widely adopted pedagogy in Singapore primary mathematics teaching. **It is extensively used**

in teaching word problems that require reasoning with fractions, ratios, and percentages. Using this approach, students use rectangular bars to visualize mathematical relationships, facilitating comprehension of abstract quantities. The model method provides a visual framework that supports students' conceptualization of fractions by illustrating the structure of mathematical relationships in word problems. Table 1 illustrates the part-whole and comparison models, the two key relational concepts that underpin the model method.

Table 1: Part-whole model for multiplication and division & multiplicative comparison models (Kho et al., 2014, p. 277)

The following part-whole model represents a whole divided into 3 equal parts:  The model illustrates the concept of multiplication as: One part × Number of parts = Whole	Part-whole model: The total is determined by the multiplication of one part and the number of parts. Conversely, if we know the total and one factor, we can find the other factor through division.
	Comparison model: The larger quantity is three times the smaller quantity, and conversely, the smaller quantity is one-third of the larger quantity. For example, if the larger quantity represents three units, the smaller quantity represents one unit. Together, they total four units, with a difference of two units between them.

2.2 Unitary Method

Complementing the model method, the unitary approach provides a structured strategy that enables learners to solve problems by first identifying the value of a single unit (part) and then scaling it to determine the value of multiple units (Fong, 1999). Though not included as part of Singapore's official mathematics syllabus, this strategy is often integrated into classroom teaching to support conceptual understanding and problem-solving involving fractions, ratios, proportions, and percentages.

2.3 Structure-Mapping Theory

Structure Mapping Theory (SMT), developed by Gentner (1983), provides a theoretical framework for understanding how learners make sense of new information through analogy and relational reasoning. SMT posits that learners interpret analogies by mapping structural relationships between a familiar base domain and a less familiar target domain. When students are presented with both a flawed and a correct bar model, the **incorrect solution often serves as the** *familiar base domain*, reflecting their intuitive reasoning or prior misconceptions. In contrast, the **correct solution acts as the** *less familiar target domain*, representing a structurally sound, yet less readily grasped, interpretation of the problem. By mapping the relational structure between these two models, students are prompted to **identify correspondences and structural mismatches**, such as misaligned quantities or misinterpreted part-whole relationships. Importantly, this comparison serves not just to reveal errors but to refine reasoning. The process shifts student focus away from surface features to **underlying relationships between parts and wholes**, or between quantities and their units.

2.4 Bruner's Modes of Representations

Bruner's (1966) Theory of Representation frames this study's approach to using bar models as visual tools that support learners in moving from enactive experiences to symbolic reasoning. Positioned within the iconic mode, bar models enable students to visualise mathematical relationships and transition toward formal expressions. Digital interactivity and structured dialogue enhance this progression, illustrating how Bruner's modes interact dynamically in the development of mathematical understanding.

2.5 Vygotsky's Sociocultural Theory

This study draws on **Vygotsky's sociocultural theory** (Vygotsky, 1978), which emphasizes the social nature of learning and the importance of cultural tools in the development of higher mental functions. The theory asserts that cognitive development occurs through **social interaction** and the use of **cultural tools** that mediate thinking.

2.5.1 Zone of Proximal Development (ZPD)

Vygotsky (1978) viewed learning as a fundamentally social process, emphasising the Zone of Proximal Development (ZPD) as the space where learning occurs through guided interaction. The ZPD is the gap between what a learner can do alone and with support, representing the optimal zone for learning through challenge and guidance from a more knowledgeable other (e.g., teacher, peer, or adult). Vygotsky proposed that learning begins through social interaction, with knowledge co-constructed interpersonally, and is later transformed into individual understanding through an "intrapersonal" (p. 131) process of internalization. Children develop understanding by engaging with more knowledgeable others through dialogue, questioning, and feedback. These interactions are not just supportive, they are the very **mechanism of cognitive growth.**

2.5.2 Cultural Tools

Vygotsky highlighted the role of **cultural tools** in shaping cognitive development, emphasizing that learning is not just about acquiring information but about **appropriating the tools of one's culture** to think and reason in new ways. In this study, the **bar model** serves as a **cultural tool** that scaffolds reasoning when introduced, modelled, and practiced within a social context. By representing part-whole, comparison, or unitary relationships, students engage with a shared system that helps them visualize and understand abstract concepts. More importantly, addressing misconceptions through dialogic interaction, questioning, and feedback enabled students to take the lead in their learning process, fostering deeper understanding and critical reasoning.

2.6 Formative Assessment Framework

Black and Wiliam (2009) describe formative assessment as an interactive process in which evidence of student thinking is continuously used to adjust instruction. This approach shifts assessment from end-point evaluation to a tool for shaping learning as it happens, guided by five core strategies:

1. Clarifying learning intentions and criteria for success
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding
3. Providing feedback that moves learners forward
4. Activating students as instructional resources for one another
5. Activating students as the owners of their own learning

This study's design was shaped by Black and Wiliam's framework, with tasks developed to reflect core formative assessment strategies. For example, learning intentions were clarified through word problems that explicitly defined the visual reasoning goal of each task (Strategy 1). Teachers implemented bar model tasks to initiate discussion and uncover students' reasoning by having them construct and analyse visual representations (Strategy 2). The tasks were designed to support student dialogue around bar models, making student reasoning visible for the teacher to respond with appropriate feedback (Strategy 3). The tasks were structured to invite peer critique and collaborative meaning-making with the aim of activating them as learning resources for one another (Strategy 4). Lastly, the opportunity to construct, revise, and interpret their own models supported students in monitoring their own thinking, thus **encouraging ownership of learning** (Strategy 5). Through this alignment, the task design served as both a **diagnostic tool and a learning scaffold**, allowing the teacher to observe conceptual development, adapt instruction, and guide the class toward deeper understanding based on the evidence generated through interaction.

3. Research Questions

This study seeks to examine formative assessment and feedback through digitally supported mathematical experiences. The research questions guiding the study are as follows:

RQ1. In what way can comparison of correct and incorrect solutions be used as a formative assessment strategy to identify students' understanding and misconceptions of mathematical concepts?

RQ2. How does the use of solution comparisons in formative assessment help teachers identify learning needs and inform instructional planning?

4. Methods

4.1 Research Design

This study employed a qualitative research design aimed at exploring how teachers use student comparisons of correct and incorrect solutions as a formative assessment tool to inform task redesign. The focus was on capturing the reasoning processes of teachers as they interpreted student responses and considered instructional adaptations, rather than measuring the direct outcomes of the modified tasks.

4.2 Participants and Context

This study was conducted in a Grade 6 mathematics classroom at a Czech elementary school. The class consisted of 9 students aged 12 with varying levels of proficiency in the target topic, fractions. While the sample size (n = 9) may appear limited, it was selected based on practical considerations, including access to participants and the context of the study. Moreover, given the specific focus of the research, a smaller sample was sufficient to gather rich data that addressed the research objectives. The depth of data collected from these participants provides sufficient grounding for the analytical claims made in the paper. Additionally, although curriculum content may differ across schools, students are generally expected to develop fluency in fractional reasoning by exploring part-whole relationships, performing operations with fractions, and interpreting representations in various forms, including ratios, decimals, and percentages. The researcher also served as the teacher during the intervention. This dual role provided an opportunity for in-situ observation and interaction with students as they engaged with the tasks. Using the Jaccard Index (Real & Vargas, 1996), intercoder agreement was measured at 0.769 (76.9%), indicating substantial agreement (Landis & Koch, 1977). While the teacher-researcher role may have inherent biases, the involvement of independent intercoders ensured that any influence from the teacher's perspective was minimized, supporting the reliability of the coding process.

4.3 Intervention Design

The intervention consisted of a structured teaching sequence (Figure 1) spanning five 45-minute lessons organised into 3 phases. Phase One comprises an orientation session introducing students to the app's bar model interface, which they engaged with interactively to explore concepts, manipulate visual representations, and deepen their understanding of part-whole and comparison relationships. *Student understanding was assessed through a post-session quiz (Quiz 1), during which* the students' on-screen activities were recorded.

Phase Two, incorrect solutions were compiled from the recordings of Quiz 1 using screenshot captures. As part of the side-by-side comparison activity, an incorrect solution was selected and paired with a corresponding valid solution created by the researcher. Both solutions were displayed alongside each other on the class screen to engage students in collaborative analysis and discussion. The observation was documented to capture students' engagement with the comparison task. Figure 2 presents coded excerpts of student dialogue, illustrating the nature of their reasoning and peer interactions.

In the final phase, a second quiz was administered to assess students' learning progression. Digital data from Quiz 1 and Quiz 2 were processed in the Atlas.ti programme using a mixed-methods framework to identify meaningful patterns and support the development and refinement of coding categories.

Figure 1: Research design sequence

4.4 Data Collection and Data Analysis

The dataset analysed in this study was originally collected as part of a classroom-based project (Poh & Jančařík, 2025) (Figure 2). Preliminary quiz data from the first study were used to inform initial coding categories. The initial analysis from that study is currently under review. While the earlier study examined comparison as a strategy for teaching fraction, the same dataset is re-analysed through a different lens to explore comparison as an interpretative framework for formative assessment. A total of approximately 200 minutes of digital data and 45 minutes of recorded sessions were analysed to identify and categorize students' incorrect solutions and misconceptions. The analysis focused on Session 3, where students jointly explored and discussed the bar model solutions displayed in Table 2. Student dialogues and digital interactions were analysed as core data. The analytical process followed an inductive approach guided by partially pre-defined categories (Dey, 1999; Strauss & Corbin, 1990). As a first step, the digital transcripts were categorized by notable episodes that reflected critical instances of reasoning or interaction. As shown in Table 3, the episodes were assigned V-type codes, referring to visual attributes of the bar models and analysed in relation to the strategies reflecting part-whole reasoning, comparison concept and unitary method. These strategies were assigned C-type codes.

Table 3: Visualisation and conceptualisation coding schemes with descriptions

Coding Scheme	Coding Category	Description
		Students referring to...
Code V (Visualisation)	V-RP (partitioning into equal units)	the partitioning on the bar model (BM)
	V-RA (alignment)	the alignment of the partitions on the BM
	V-RSM (size of bar model)	the size (length) of the BM
	V-RS (shaded part)	the shaded part of the BM
	V-RL (labelling of units)	the labelling of units on the BM
		Students coping with the task by...
Code C (Conceptualisation)	C-PW (part-whole reasoning)	exploring part-whole reasoning
	C-C (comparison concept)	using comparison concept
	C-U (unitary method)	using unitary method

Two quizzes were used to track learning: Quiz 1 at the start and Quiz 2 at the end of the study (Figure 3). Statistical analysis showed a positive trend: seven of nine students improved, one remained the same (P2), and one declined slightly (P5). Mean scores increased from 57.1% to 59.6%. A paired t-test confirmed a significant gain (p = 0.033) with a large effect size (Cohen's d = 0.86). These findings suggest a positive impact but should be interpreted with caution due to the small sample size.

T	S/ R	Talk Turn	C-Codes	V-Codes
18	R	What is the difference between the two solutions?		
19	S3	This (RHS)... the two are (the) same long (length), this (LHS) ... (they have) longer and shorter (lengths).	C-C	V-RSM
20	R	Can someone tell me which solution is right and why.		
21	S8	This one (RHS) is right … because … it (the question) is "same amount left".	C-C	V-RSM; V-RS
22	R	Which section of the bar says, "same amount left"?		
23	S8	Here (RHS) ... (the) same length ... (the) first (upper bar) and (the) second (lower bar)	C-C	V-RA; V-RSM; V-RS
24	R	Do you mean the whole length or the blue parts (RHS)?		
25	S3	The blue part (RHS).	C-C	V-RSM; V-RS
26	R	The blue parts (RHS), ok. Why?		

27	S3	(RHS) Because “same amount left”.	C-C	V-RSM; V-RS
32	R	(RHS) So we know all the units are equivalent to $170. We need to find the value of one unit. Why?		
33	S7	(RHS) Find if how many units Maria (has), also Jan and how many units spent and how many units left...everything.	C-U; C-PW; C-C	V-RP; V-RA; V-RL
34	R	(RHS) Yes, if we know the value of one unit, we can calculate all the values. How do we find one unit?		
35	S2	(RHS) $170 divide (by) all the units.	C-U	V-RP
44	R	(RHS) Can you use another way to find the value of one unit? Can you see two units are equivalent to $10?		
45	S2	(RHS) $10 divide (by) 2 is $5...10 units times $5 is $50 ...Oh...	C-U; C-C; C-PW	V-RP
46	R	Ok. This (RHS) does not work out to $170. Let’s look at the other solution (LHS). What’s the difference?		
47	S3	(LHS) First one (upper bar) is longer and under (lower bar) is shorter	C-C	V-RA; V-RSM
48	R	(LHS) That means Maria has more money than Jan. How do we find one unit here?		
49	S2	(LHS) $170 divide (by) all the units.	C-U	V-RP
50	R	(LHS) Can you tell me exactly how many units?		
51	S8	(LHS) 3 units plus 3 units plus 2 units...8 units.	C-U	V-RP;
52	S2	(LHS) And plus $10.	C-C; C-PW	V-RS; V-RA, V-RL
53	S8	(LHS) Ok...$170 divide (by) 8 units plus $10.	C-U; C-C; C-PW	V-RS; V-RA, V-RL

***Legend**: T Turn: S/ R: Student/ Researcher; LHS: Left Hand Side; RHS: Right Hand*

Figure 2: A coded transcript of talk turns during side-by-side comparison of solutions

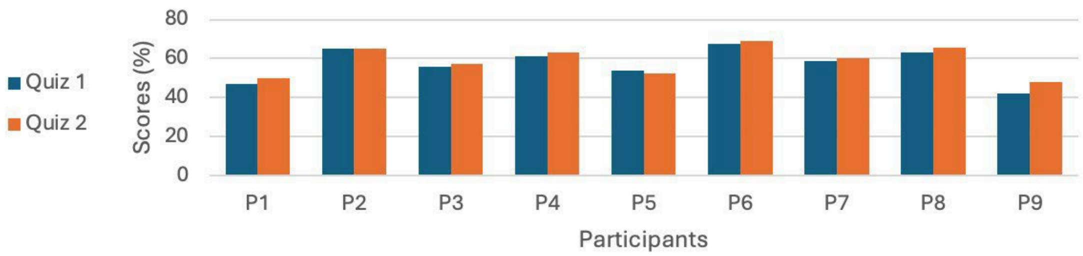
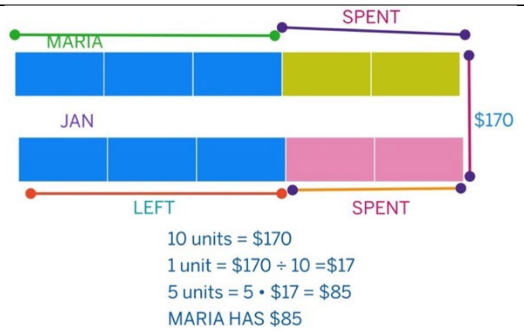


Figure 3: Student Performance (Quiz 1 vs Quiz 2)

5. Results and Implications

The following results emerged from a detailed analysis of qualitative data, supported by quantitative trends where applicable, in response to the guiding research questions.

Table 2: Comparison of valid and incorrect bar model solutions

<table>
<tr><td colspan="2">Maria and Jan have $170 altogether. If Maria spends $\frac{2}{5}$ of her money and Jan spends $10, then they will have the same amount left. How much money does Maria have?</td></tr>
<tr><td>Suggested Solution (LHS)</td><td>Filip's Solution (RHS)</td></tr>
<tr><td>
</td><td>
</td></tr>
</table>

***Legend**: LHS: Left Hand Side; RHS: Right Hand Side*

Sections 5.1 and 5.2 provide the basis for insights into RQ1: **In what way can comparison of correct and incorrect solutions be used as a formative assessment strategy to identify students' understanding and misconceptions of mathematical concepts?**

5.1 Use of Visual Representations to Support Comparison, Part-whole and Unitary Reasoning

5.1.1 Reasoning with comparison concept

Students demonstrated awareness of the comparison concept by interpreting visual representations that highlighted differences and relationships between quantities. When comparing amounts in the word problems, students showed an understanding of relational structure by identifying who had more, less, or the same (T19, T21, T23, T25, T27, T47). The comparison concept was evident as the student analysed the initial amounts, expenditures, and remaining quantities for both Maria and Jan, making relational judgments between them (T33, T52, T53).

5.1.2 Interpreting Part-whole Relationship

Part-whole understanding was evident the student's ability to identifying how amounts spent and saved relate to the total amount (T33, T52, T53). This demonstrated an awareness that the whole is composed of distinct but related parts, and that reasoning about either part could be used to infer or verify the whole.

5.1.3 Reasoning with Unitary Structure in Visual Representations

Students demonstrated an understanding of unitary structure through their ability to identify a known total ($170 – $10) associated with a specific number of equal parts (8 parts). From this, they used division to determine the value of a single unit, which they then multiplied by the required number of units to solve the problem (T35, T49, T51, T53). This process reflected flexible reasoning and an ability to navigate between parts and wholes using multiplicative strategies.

5.2 Correction of Error Through Collaborative Discussion

By comparing both correct and incorrect bar model solutions, students were prompted to articulate and justify their reasoning, which exposed misconceptions (T45). Students' capacity to reflect on and revise their reasoning was observed during a task where an initial misunderstanding emerged. The group had misinterpreted the **equal bar lengths** as indicating that *Maria and Jan had the same total amount of money* (T21), rather than recognising that the bars represented the **same amount remaining** after spending. Although they correctly noted that the "blue parts" (T25) of the bar model were equal—representing the money left—the critical distinction between what was *left* versus what was *originally had* was not initially clear. As they engaged in further discussion, the students began to question their initial assumption. This collaborative dialogue involved **justifying their interpretations, recalculating unit values**, and **re-examining the structure** of the model. Their joint reasoning led to the recognition that the original amounts of money could not have been equal, as the amounts spent were different. This shift in understanding was reflected in their rejection of the initial model and the development of a corrected solution (T27, T33, T35).

5.3 Pedagogical Implications

The collaborative discourse illustrates how student discussion provided a platform for conceptual clarification, enabling them to correct their misconceptions. This highlights the importance of structuring classroom activities that promote collaborative inquiry, where students actively engage in shared problem-solving. Such interactions not only enhance mathematical reasoning but also encourage students to take ownership of their learning, ultimately improving their ability to apply concepts in novel contexts.

5.4 Leveraging Formative Insights for Instructional Design

Section 5.4 provides evidence-based responses to RQ2, addressing how teachers can act on these findings to refine their instructional approaches: How does the use of solution comparisons in formative assessment help teachers identify learning needs and inform instructional planning?

5.4.1 Eliciting Learning Needs Through Student Thinking

One key learning need identified in this task was the misidentification of structural equivalence (T45). Students appeared to confuse which quantity was equal across the two individuals, mistaking equal remainders for equal total amounts and consequently misrepresented this relationship in their bar models. While the visual models were intended to support reasoning (T23, T25, T27, T33), some students relied on surface features such as equal bar lengths, without attending to the relational meaning of the parts and wholes represented (T35, T45, T49, T51).

5.5 Didactic implications

The task highlighted a learning gap, suggesting a deeper issue with representational reasoning, where students may recognise individual quantities but struggle to coordinate them meaningfully within a part-whole structure. Specifically, they need greater support in distinguishing **what is being compared**, for instance, total amounts, amounts spent, or amounts remaining and how these values relate to one another in context.

5.5.1 Informing Pedagogical Practice

To address this learning need, instructions should include tasks that contrast different types of equivalence (e.g., equal totals vs. equal remainders), prompt students to describe key visual features of bar models (Table 3) and explain their reasoning. Peer comparison of alternative models (T45, T47) can help surface misunderstandings and support more accurate interpretations. One effective strategy is to design tasks with intentional ambiguity, requiring students to reason through the specific equivalence and reflect this accurately in their bar model. For example, students might be presented with the following question (Figure 4):

Maria and Jan each spent $\frac{3}{4}$ of their money. Maria had $80 at first *and* Jan had $60. *How much money do they have left altogether?*

This example is not a result of the study, but a design decision shaped by insights gathered during classroom observation. It demonstrates how the study's insights are operationalised in practice, bridging empirical observation with pedagogical action. The intended ambiguity in this task lies in the fact that the **same fraction refers to the portion spent**, rather than the total amount or the amount remaining. While both Maria and Jan spent $\frac{3}{4}$ of their money, the actual amounts differ due to their different original totals. This subtle distinction encourages students to think critically about what the bar model represents and reason proportionally, rather than assuming equivalence in absolute terms. Given that many bar model tasks typically emphasize equal totals or equal remainders, students may default to these more familiar interpretations. As a result, they may mistakenly assume that equal bar lengths indicate equal total amounts or remaining values, rather than correctly understanding that same fraction ($\frac{3}{4}$) refers to the portion spent, not identical quantities, as opposed to the previous scenario. This creates a productive tension that encourages deeper reasoning about part-whole relationships and the structure of the problem.

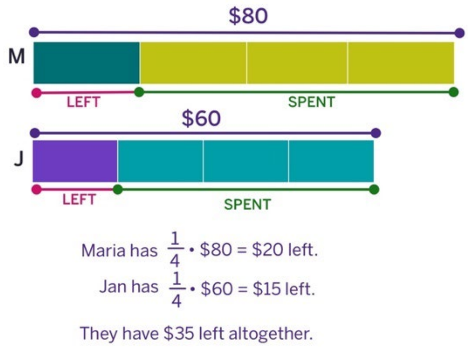


Figure 4: Suggested post-intervention task prompt

6. Discussion

This study offers pedagogical insights that students' constructions revealed both conceptual strengths and common misconceptions. While some students struggled with representation, others used units to reason effectively, highlighting bar models' role in externalising students' thought processes in ways that are accessible to both peers and teachers. Teachers can draw on these insights to inform instructional approaches through visual comparison tasks, suggesting that purposeful task design can target specific conceptual hurdles, including the misidentification of structural equivalence. Although the current implementation was shaped by a specific classroom context, the underlying pedagogical principles, such as using student-generated visual models and structured student discussion within a digitally mediated environment are adaptable to a range of school environments, student backgrounds and prior knowledge. Reflecting on this adaptability can inform efforts to embed such practices into standard classroom routines, supporting scalability without compromising pedagogical integrity.

7. Limitations

As with all research, this study has certain limitations that should be noted. Because the findings were closely tied to the teaching design, students' reasoning was shaped within the structure of the learning activities. This may have constrained the opportunity to observe students' spontaneous application of the concepts. Future studies could adopt a longitudinal approach to examine how sustained use of these technologies affects students' mathematical reasoning over time, including their ability to construct and critique models independently. Additionally, the use of group settings instead of pairs could have altered interaction patterns, possibly affecting the depth of individual engagement. Future studies could investigate whether findings hold across different group structures. Further, while students demonstrated conceptual understanding in discussions, quiz results may reflect technical difficulties with the unfamiliar app rather than actual learning gaps. Alternative assessment methods such as longitudinal observations or performance-based tasks might better capture students' conceptual progress.

8. Conclusion

This study highlights how digital assessment facilitates new forms of engagement for student thinking and reasoning as well as instructional practices and teacher response. When students engage with visual models, their thinking becomes visible in ways that support both learning and teaching. These interactions provide valuable diagnostic insights, helping teachers identify conceptual misunderstandings and deliver targeted instruction. Visual representations also create a platform for meaningful student-led dialogue, where reasoning can be explored collaboratively. Such practices promote deeper mathematical understanding and align with key principles of formative assessment.

Ethics Declaration

Ethical clearance was not required. The study occurred in regular classroom instruction without sensitive data. Responses were anonymised with pseudonyms, and no identifying information about schools or participants is disclosed.

AI Declaration

All content and analyses are the author's own with minor AI-assisted language refinement.

References

Black, P., & Wiliam, D. (2009). Developing the Theory of Formative Assessment. *Educational Assessment, Evaluation and Accountability*, *21*(1), 5–31. https://doi.org/10.1007/s11092-008-9068-5

Bruner, J. S. (1966). *Toward a Theory of Instruction*. Belknap Press of Harvard University.

Dey, I. (1999). *Grounding Grounded Theory : Guidelines for Qualitative Inquiry*. Academic Press.

Drijvers, P., Ball, L., Barzel, B., Heid, M. K., Cao, Y., & Maschietto, M. (2016). Uses of Technology in Lower Secondary Mathematics Education. In *ICME-13 Topical Surveys* (1st ed., pp. 1–34). Springer International Publishing. https://doi.org/10.1007/978-3-319-33666-4

Fong, H. K. (1999). Strategic Models for Solving Ratio and Proportion Problems. *The Mathematics Educator*, *4*(1), 34–51. The NIE Digital Repository. https://repository.nie.edu.sg/entities/publication/ed046a27-97b6-4a42-917d-d01593833aff/details

Gentner, D. (1983). Structure-Mapping: A Theoretical Framework for Analogy. *Cognitive Science*, *7*(2), 155–170. https://doi.org/10.1207/s15516709cog0702_3

Kho, T. H., Yeo, S. M., & Fan, L. (2014). *Model Method in Singapore Primary Mathematics Textbooks*. 29–31. University of Southampton.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review*, *92*(1), 109–129. https://doi.org/10.1037/0033-295x.92.1.109

Landis, R. J., & Koch, G. G. (1977). The Measurement of Observer Agreement for Categorical Data. *Biometrics*, *33*(1), 159–174. https://doi.org/10.2307/2529310

Nesher, P., Greeno, J. G., & Riley, M. S. (1982). The Development of Semantic Categories for Addition and Subtraction. *Educational Studies in Mathematics*, *13*(4), 373–394. https://doi.org/10.1007/bf00366618

OECD (2016) Innovating Education and Educating for Innovation*: The Power of Digital Technologies and Skills, Educational Research and Innovation.* OECD Publishing. https://doi.org/10.1787/9789264265097-en

Poh, C., & Jančařík, A., (2025). Side-by-side comparison of bar models as a strategy for teaching fractional relationship in word problems. In: J. Novotná & H. Moraová, *Elementary Mathematics: Building Strong (SEMT 2025),* pp. 352–362.

Real, R., & Vargas, J. M. (1996). The Probabilistic Basis of Jaccard's Index of Similarity. *Systematic Biology*, *45*(3), 380–385. https://doi.org/10.1093/sysbio/45.3.380

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2017). The Power of Comparison in Mathematics Instruction: Experimental Evidence from Classrooms. *Acquisition of Complex Arithmetic Skills and Higher-Order Mathematics Concepts*, 273–295. https://doi.org/10.1016/b978-0-12-805086-6.00012-6

Strauss, A., & Corbin, J. (1990). *Basics of Qualitative Research: Grounded Theory Procedures and Techniques.* Sage

Vygotsky, L. (1978). *Mind in society: The Development of Higher Psychological Processes*. Harvard University Press.